Fast EM Scattering Analysis for the Hard Targets in a Layered Medium by Using the Hierarchical Vector Basis Functions

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Abstract **─** This paper presents an efficient algorithm combining a higher order method of moments (MoM) with the adaptive cross approximation-singular value decomposition (ACA-SVD) algorithm for the three-dimensional perfect electric conductor (PEC) targets located in a planar layered medium. An efficient set of hierarchical divergence-conforming vector basis functions based on curved triangular mesh is used to expand the induced surface electric current density on the metal surface of the target in layer medium. A three level discrete complex images methods (DCIM) is applied to efficiently obtain the closed form spatial Green's functions. To be able to solve the electrically large problems, the ACA-SVD algorithm is proposed to accelerate the matrix-vector multiplication and reduce the memory requirements when the corresponding matrix equation is solved by a Krylov-subspace iterative method. The electromagnetic scattering from the practical model of hard targets buried in a lossy medium is analyzed in this paper, and numerical examples demonstrate the accuracy and efficiency of the proposed technique for the electrically large scattering problems in layered medium.

Index Terms **─** Complex electromagnetic scattering, hierarchical vector basis functions, modified adaptive cross approximation (ACA) algorithm.

I. INTRODUCTION

The development of an efficient method for computing electromagnetic scattering from targets located in layered medium has received intense attention during the past decades because of its important applications in many areas, such as, remote sensing, target identification, geophysical exploration, microwave imaging, and etc. [1]-[3]. Even so, the fast and accurate detection of arbitrary shaped electrically large hard targets in a multilayered media is still a challenging problem, and only very few of papers which truly deal with the electrically large electromagnetic scattering problems in a multilayered media were published in the past years [1]-[2]. The method of moments (MoM) [4] is one of the most widely used techniques for solving electromagnetic scattering problems since they require fewer unknowns than differential equations solvers. Traditional MoM solution of the surface integral equation (SIE) utilizes the Rao-Wiltion-Glisson (RWG) [4] or rooftop basis functions [5] that based on flat triangular or quadrilateral patches to expand the surface current density, which belongs to the low-order method. When the electrical size of the problem is very large, plane triangles used to discrete the surface will produce a large number of unknowns, but cannot provide enough flexibility and efficiency in modeling curved structures. Further more, the accuracy of solution while using the low-order bases is improved slowly with increases in the number of unknowns. By using higher order basis functions which are defined on curved elements to expand the unknowns, the higher order method [6] essentially reduces the computational unknowns, and enhances the accuracy of the solution at the same time. Higher order MoM solutions of the SIE have been successfully used to solve electrically large electromagnetic problems in free space [7]-[9]. Jorgensen [3] also uses the higher order technique to analyze the electromagnetic scattering problems for metallic objects in layered media, but hasn't combined any fast algorithm.

With N being the number of unknowns of the MoM solution, the computational complexity is $O\left(N^2\right)$ when an iterative method is applied. To reduce the memory requirement and speed up the matrix-vector multiplication, fast algorithms should be involved. One of the most popular techniques is fast mulipole method (FMM) [10] or multilevel fast mulipole algorithm (MLFMA) [11]-[14]. The MLFMA is based on the vector addition theorem and the plane wave expansion theory, which means the formulation, implementation, and occasionally performance of the MLFMA depends on a priori knowledge of Green's functions. Therefore, when the EM scattering problem is discussed in layered media, the MLFMA cannot be directly applicable due to the more complicated Green's functions. Fast fourier transform (FFT) based algorithms also have been used to accelerate the analysis of scattering from objects in

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layered media [1]-[2], and the parallel implementation of the AIM was used to deal with the electrically large scattering problems in [1]. In contrast with the MLFMA and FFT based algorithms, the low-rank approximation based adaptive cross approximation (ACA) algorithm [15] is purely algebraic and, does not depend on the formulation of Green's functions. In general, the ACA requires considerable computation time and memory. To improve the efficiency, the ACA-SVD was studied by Bebendorf [16], which efficiently recompresses the matrices of ACA using the SVD technique. In this paper, the ACA-SVD technique is presented for solving electromagnetic problems in layer medium.

This paper is organized as follows. In Section II, we present the higher order hierarchical vector basis functions and the modified ACA formulations for electrically large scattering problems in layered medium. A three level discrete complex images methods (DCIM) is applied to efficiently obtain the closed form spatial Green's functions without any quasi-static and surface-wave extraction, that part has been reported in the conference paper [17]. A set of hierarchical divergence-conforming vector basis functions based on curvilinear triangle elements are used to expand the induced current density on the surface of targets in layered medium. The ACA-SVD technique is employed to reduce the memory requirements and computational complexity. In Section III, we present several results to verify the proposed method's accuracy and efficiency. In this section, the computational complexity and memory requirements of the presented method are discussed and compared with existing fast algorithms, the adaptive integral method (AIM), for electrically large scattering problems in layered medium. Finally, conclusions are summarized in Section IV.

II. THEORY AND FORMULATION

A. Integral equation in layered medium and higher order MoM formulation

Consider an arbitrarily shaped 3-D perfect electric conductor (PEC) targets located in the pth layer of a layered medium, as shown in Fig. 1. The background medium has N parallel layers with independent permittivity and permeability (ε_p, μ_p, $p = 1, \cdots, N$). Consider a general purpose scattering problem of a time-harmonic electromagnetic field $\mathbf{E}^{inc}$, incident on the PEC structure. Using the continual boundary condition for the tangential component of the total electric field on the PEC surface, we formulate the mixed potential electric field integral equation (MPIE) [18], as shown:

$$\hat{\mathbf{n}} \times \left[j\omega \mathbf{A}(\mathbf{r}) + \nabla \phi(\mathbf{r}) \right] = \hat{\mathbf{n}} \times \mathbf{E}^{inc}(\mathbf{r}). \tag{1}$$

The vector magnetic potential $\mathbf{A}(\mathbf{r})$ and the scalar electric potential $\phi(\mathbf{r})$ due to the surface current density $\mathbf{J}(\mathbf{r}')$ can be expressed as:

$$\mathbf{A}(\mathbf{r}) = \int_{S'} \bar{\mathbf{G}}^A(\mathbf{r}, \mathbf{r}') \bullet \mathbf{J}(\mathbf{r}') dS', \tag{2}$$

$$\phi(\mathbf{r}) = \int_{S'} K^\phi(\mathbf{r}, \mathbf{r}') \nabla' \bullet \mathbf{J}(\mathbf{r}') dS', \tag{3}$$

where $\mathbf{r} = \rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}$ and $\mathbf{r}' = \rho'\hat{\boldsymbol{\rho}} + z'\hat{\mathbf{z}}$ denote observer and source points, respectively. $\bar{\mathbf{G}}^A(\mathbf{r}, \mathbf{r}')$ is the dyadic green function and $K^\phi(\mathbf{r}, \mathbf{r}')$ is the scalar Green's functions in layered medium. These functions are derived using formulation C by Michalski and Zheng [15], and the traditional form of $\bar{\mathbf{G}}^A(\mathbf{r}, \mathbf{r}')$ and $K^\phi(\mathbf{r}, \mathbf{r}')$ are given by:

$$\bar{\mathbf{G}}^A(\mathbf{r}, \mathbf{r}') = (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}) G^A_{xx} + \hat{\mathbf{z}}\hat{\mathbf{x}} G^A_{zx} + \hat{\mathbf{z}}\hat{\mathbf{y}} G^A_{zy} + \hat{\mathbf{z}}\hat{\mathbf{z}} G^A_{zz}, \tag{4}$$

$$K^\phi(\mathbf{r}, \mathbf{r}') = K^\phi(\rho; z|z'), \tag{5}$$

where

$$\begin{aligned} G^A_{xx}(\rho; z|z') &= S_0\left\{\tilde{G}^A_{vv}(k_\rho; z|z')\right\}, \\ G^A_{zz}(\rho; z|z') &= S_0\left\{\tilde{G}^A_{zz}(k_\rho; z|z')\right\}, \\ G^A_{zx}(\rho; z|z') &= -j\cos\varphi S_1\left\{\tilde{G}^A_{zu}(k_\rho; z|z')\right\}, \\ G^A_{zy}(\rho; z|z') &= -j\sin\varphi S_1\left\{\tilde{G}^A_{zu}(k_\rho; z|z')\right\}, \\ K^\phi(\rho; z|z') &= S_0\left\{\tilde{K}^\phi(k_\rho; z|z')\right\}. \end{aligned} \tag{6}$$

S_n denotes Sommerfeld integral (SI) of order n and is defined as:

$$S_n\left\{\tilde{f}(k_\rho)\right\} = \frac{1}{2\pi}\int_0^\infty \tilde{f}(k_\rho) J_n(k_\rho\rho) k_\rho dk_\rho, \tag{7}$$

J_n is the Bessel function of order n. The three level discrete complex images methods (DCIM) proposed in [17] are used to calculate the SIs and obtain the closed form spatial Green's functions. The three levels DCIM can approximate the contribution of both the lateral waves and surface waves, and it is more stable and accurate than traditional method for the lossy stratified media [17].

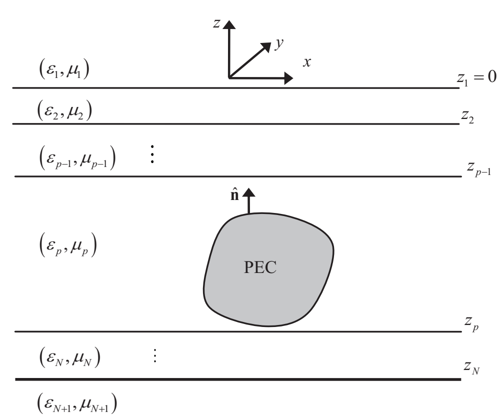


Fig. 1. Arbitrarily shaped 3-D PEC target in a layered medium.

To solve the Equation (1), the surface of scatterer should be discreted, and the induced current density should be expanded by basis functions. The PEC body in layered media is meshed using cured triangular patches of second order with associated parametric curvilinear coordinate systems defined by $0 \le \xi_1, \xi_2, \xi_3 \le 1$. The efficient higher order (HO) hierarchical divergence-conforming vector basis functions defined on curvilinear triangular patches used here were proposed in [9], and that have been proved could provide well-conditional linear system for iterative solution. For sake of simplicity, we only give the expression of hierarchical basis functions of order 1.5. The rest higher order's expressions of these basis functions can be found in [9]. The hierarchical basis functions of order 1.5 for one curved triangular element are defined as:

$$\begin{aligned}
&\mathbf{f}^e_{1,0}(\mathbf{r}') = \frac{1}{J}\left[\left(\xi_1 - 1\right)\frac{\partial \mathbf{r}'}{\partial \xi_1} + \xi_2 \frac{\partial \mathbf{r}'}{\partial \xi_2}\right], \quad \mathbf{f}^e_{1,1}(\mathbf{r}') = \sqrt{3}\left(\xi_2 - \xi_3\right)\mathbf{f}^e_{1,0}(\mathbf{r}'),\\
&\mathbf{f}^e_{2,0}(\mathbf{r}') = \frac{1}{J}\left[\xi_1 \frac{\partial \mathbf{r}'}{\partial \xi_1} + \left(\xi_2 - 1\right)\frac{\partial \mathbf{r}'}{\partial \xi_2}\right], \quad \mathbf{f}^e_{2,1}(\mathbf{r}') = \sqrt{3}\left(\xi_3 - \xi_1\right)\mathbf{f}^e_{2,0}(\mathbf{r}'),\\
&\mathbf{f}^e_{3,0}(\mathbf{r}') = \frac{1}{J}\left[\xi_1 \frac{\partial \mathbf{r}'}{\partial \xi_1} + \xi_2 \frac{\partial \mathbf{r}'}{\partial \xi_2}\right], \quad \mathbf{f}^e_{3,1}(\mathbf{r}') = \sqrt{3}\left(\xi_1 - \xi_2\right)\mathbf{f}^e_{3,0}(\mathbf{r}'),\\
&\mathbf{f}^f_{1,01}(\mathbf{r}') = 2\sqrt{3}\xi_1 \mathbf{f}^e_{1,0}(\mathbf{r}'), \quad \mathbf{f}^f_{2,01}(\mathbf{r}') = 2\sqrt{3}\xi_2 \mathbf{f}^e_{2,0}(\mathbf{r}'),
\end{aligned} \tag{8}$$

where the superscript denotes the edge or face-based functions, the first subscript denotes the local edge number for a fixed triangular element, the second subscript of the edge-based functions and the sum of the two number of the face-based functions' second subscript are both denote order of hierarchical polynomials, the $\mathbf{r}'$ is the position vector of the source point determined by normalized face coordinated on curved parametric triangular patch, J is the element Jacobian. Obviously, the lowest order expansion (called order 0.5) is included in expression (8) (see the first three functions) and they are the well-known curvilinear Rao-Wilton-Glisson (CRWG) functions [19]. The unknown current density $\mathbf{J}\left(\mathbf{r}'\right)$ is expanded using N HO bases:

$$\mathbf{J}\left(\mathbf{r}'\right) = \sum_{n=1}^{N} I_n \mathbf{f}_n\left(\mathbf{r}'\right), \tag{9}$$

where I_n are the unknown expansion coefficients, and N equations are obtained by applying Galerkin testing method:

$$\mathbf{ZI} = \mathbf{V}. \tag{10}$$

B. ACA-SVD accelerated the higher order MoM solution

In contrast with the MLFMA and FFT based algorithms, the low-rank approximation based fast algorithms [15]-[16], [20]-[21] are purely algebraic, and do not depend on the formulation of Green's functions. An ACA based fast algorithm with recompress technique is introduced here to reduce the memory requirements and computational complexity for electrically large scattering problems in layered medium.

The ACA employs the same octree data structure as in the MLFMA. The octal-tree algorithm is used to subdivide a box that encloses an object into smaller boxes. Figure 2 shows the box enclosing the object is subdivided into smaller boxes at multiple levels, in the form of an octal-tree. The largest boxes not touching each other are at level 2, while the smallest boxes are level L. The subdivision process runs recursively until the finest level L. With reference to Fig. 3, far interactions exist at levels 2 and higher. By using the fast algorithm, the impedance matrix $\mathbf{Z}$ can be rewritten as:

$$\mathbf{Z} = \mathbf{Z}_{Near} + \mathbf{Z}_{Far}, \tag{11}$$

where the near field interactions are computed with the higher order MoM directly. Based octree grouping the far field interactions can be expressed as:

$$\mathbf{Z}_{Far} = \sum_{l=2}^{L} \sum_{i=1}^{M(l)} \sum_{j=1}^{Far(l(i))} \left[\mathbf{Z}_{lij}\right]_{m\times n}, \tag{12}$$

where $M\left(l\right)$ is the number of nonempty groups at level l and, $Far\left(l(i)\right)$ denotes the number of far interaction groups of the i-th nonempty group for each observation group $l(i)$ at level l. The $\left[\mathbf{Z}_{lij}\right]_{m\times n}$ is the sub-matrix of far interaction between the observation group i and source group j, and the subscripts m, n denote the number of the basis functions in the observation and source groups, respectively. Since the sub-matrix $\left[\mathbf{Z}_{lij}\right]_{m\times n}$ is a low τ-rank matrix, in the ACA implementation, it can be approximate to the product of two small matrices:

$$\left[\mathbf{Z}_{lij}\right]_{m\times n} = \left[\mathbf{U}_{lij}\right]_{m\times r}\left[\mathbf{V}_{lij}\right]_{r\times n}, \tag{13}$$

where r is the τ-rank of the matrix $\left[\mathbf{Z}_{lij}\right]_{m\times n}$ and is much smaller than m and n. The error in this approximation is controlled by a threshold τ, which determines when to stop looking for more columns and rows of $\mathbf{U}_{lij}$ and $\mathbf{V}_{lij}$, respectively. The ACA-SVD was presented by [11]. Since the matrices $\left[\mathbf{U}_{lij}\right]$ and $\left[\mathbf{V}_{lij}\right]$ generated by the ACA are usually not orthogonal, they may contain redundancies that can be removed by an algebraic compression technique. A SVD recompression of the obtained ACA decomposition can be performed utilizing two QR factorizations, so that either the $\left[\mathbf{U}_{lij}\right]$ or $\left[\mathbf{V}_{lij}\right]$ matrices in (13) can be done orthonormal, with an extra saving in memory, and the far field interactions in (12) can be rewritten as:

$$\mathbf{Z}_{Far} = \sum_{l=2}^{L} \sum_{i=1}^{M(l)} \sum_{j=1}^{Far(l(i))} \left[\hat{\mathbf{U}}_{lij}\right]_{m\times k}\left[\hat{\mathbf{V}}_{lij}\right]_{k\times n}, \tag{14}$$

where $k < r$. When higher order hierarchical vector basis functions are directly combined with the ACA-SVD algorithm, the higher order MoM's advantages of great reduction of the number of unknowns and acceleration of the convergence for iteration are keeping.

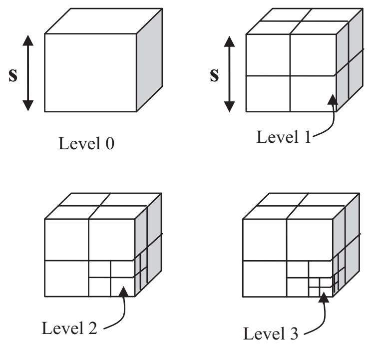


Fig. 2. Sketch of the octree structure.

III. NUMERICAL RESULTS

In this section, numerical results demonstrate efficiency of the higher order MoM solutions combined with the ACA-SVD technique for the 3D electromagnetic scattering problems in layered medium. For all computations, the threshold $\tau = 10^{-3}$. The flexible generalized minimal residual with deflated restarted (FGMRES-DR) [22] was used for the iterative solution of the system matrix. In the FGMRES-DR, all computations were carried out on a computer with 2.83 GHz CPU and 8.0 GB RAM.

The first example considers the bistatic radar cross section (Bi-RCS) of a PEC cylinder over a three-layer medium to test the accuracy of the proposed method. The three-layer medium is characterized by $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = 2.56\varepsilon_0$, $\varepsilon_3 = (6.5 - j0.6)\varepsilon_0$, $\mu_1 = \mu_2 = \mu_3 = \mu_0$. The cylinder is 3 m long and has a radius of 0.5 m, and its bottom is located 0.2 m above the top interface of the three-layer dielectric medium as shown as in Fig. 3. The incident angles of plane wave are $\theta^{inc} = 60^\circ, \phi^{inc} = 0^\circ$ at 600 MHz. This scatterer is discretized with 572 curvilinear triangular patches for order 1.5 hierarchical basis functions, and giving raises to 2860 HO unknowns. The Bi-RCS at the scattered angle $\theta^{sca} = 60^\circ$ for both VV and HH polarization was shown in Fig. 5. The HO ACA-SVD results are compared with the 3-D adaptive integral method (AIM) [1], it can be found that there is an excellent agreement among them. The memory requirement for the HO ACA-SVD and the low order (LO) ACA-SVD (which use the CRWG basis functions and curvilinear triangular discretization to make a fair comparison) are compared in the Table 1. It can be found that, the use of higher order techniques greatly reduces the memory requirement for a given problem.

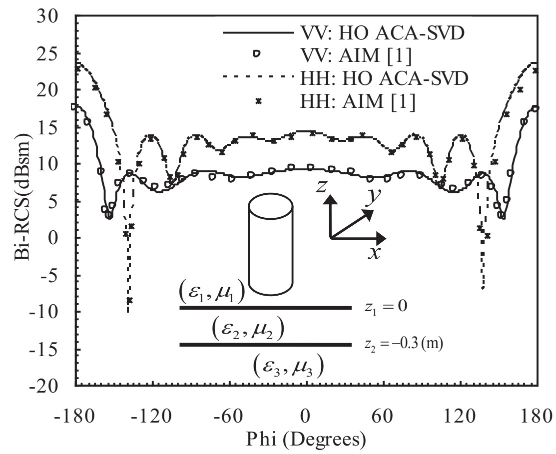


Fig. 3. Bistatic RCS of a PEC cylinder over a three-layer medium in the $\theta^{sca} = 60^\circ$ at 600 MHz.

Table 1: The memory requirement of HO ACA-SVD and LO ACA-SVD for the cylinder in layered medium

	HO ACA-SVD	LO ACA-SVD
Total number unknowns	2860	13383
DCIM time (sec.)	9.2	9.4
Near field memory (MB)	12.6	16.2
Far field memory (MB)	23.1	146.2
Matrix-vector operation time (sec.)	0.015	0.068
Total iteration steps	118	190

The second example considers the Bi-RCS of a PEC cuboid in a two-layer medium to test the efficiency of the proposed method. The two-layer medium is characterized by $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = (3.3 - j0.3)\varepsilon_0$, $\mu_1 = \mu_2 = \mu_0$. The cuboid is 5.0 m long, 5.0 m wide and 2.0 m high, and its top is located 2.5 m below the interface of the two-layer dielectric medium as shown as in Fig. 4. The incident angles of plane wave are $\theta^{inc} = 60^\circ, \phi^{inc} = -90^\circ$ at 900 MHz. The Bi-RCS at the scattered angle $\theta^{sca} = 60^\circ$ for VV polarization was shown in Fig. 4. The HO-ACA results are compared with a reference fast inhomogeneous plane wave algorithm (FIPWA) accelerated MoM [23] and the 3-D AIM [1], it can be found that there is an excellent agreement among them. This scatterer is discretized with 9144 curvilinear triangular patches for order 1.5 hierarchical basis functions, and giving rise to 45720 HO unknowns. This simulation required 3.121 GB total memory and 4941s CPU time. The CPU time of the calculation of the spatial-domain layered medium

Green's functions by DCIM is only 24s. The far-field calculation accelerated by the modified ACA-SVD required 2.973 GB memory and 4784s for filling the far-field matrices. Figure 5 shows the required memory and the CPU time as a function of the number of HO unknowns increases, respectively. The MVP refers to a matrix-vector operation in Fig. 5 (b). Increasing the frequencies of the incident wave from 300 MHz to 1.2 GHz, and fixing the electrical mesh size equal to 0.45λ, that will result in the increasing number of total unknowns. As can be observed from the Fig. 5, the total memory and the CUP time of the higher order ACA-SVD is much less than of the higher order ACA with a fixed proportion. It also can be concluded that the complexity of the higher order ACA-based algorithms are scaled as $N^{4/3}\log N$ for memory and CPU time, respectively, where N denotes the number of HO unknowns.

The third example considers the scattering of a tank model located in a four-layer medium which imitates a complex environment. The four-layer medium is characterized by $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = (5.02 - j0.2)\varepsilon_0$, $\varepsilon_3 = \varepsilon_0$, $\varepsilon_4 = (10.8 - j0.2)\varepsilon_0$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_0$. The tank model has a size 12.63 m × 4.33 m × 2.8 m as shown as in Fig. 6. The bistatic RCS was plotted at a frequency of 500 MHz and an angle $\theta^{inc} = 30^\circ$, $\phi^{inc} = 0^\circ$ for both VV polarization as shown in Fig. 7, the results are compared with the scattering for the one in free space. This scatterer is discretized with 7094 curvilinear triangular patches for order 1.5 hierarchical basis functions, and giving rise to 35470 HO unknowns. This simulation required 2.262 GB memory and 6517s CPU time. The CPU time of the calculation of the multilayered Green's functions by DCIM is only 260s. The far-field calculation accelerated by the ACA-SVD required 1.932 GB memory and 2432s for filling the far-field matrices.

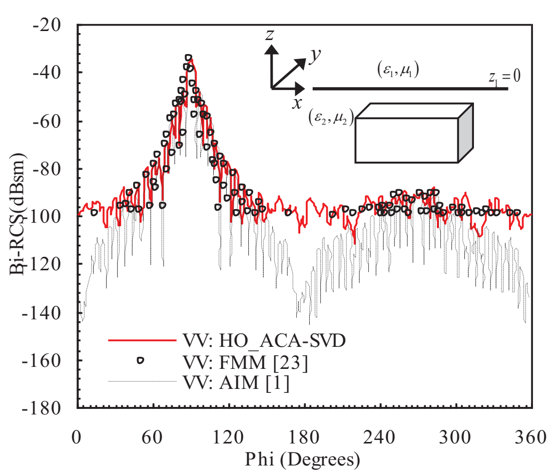
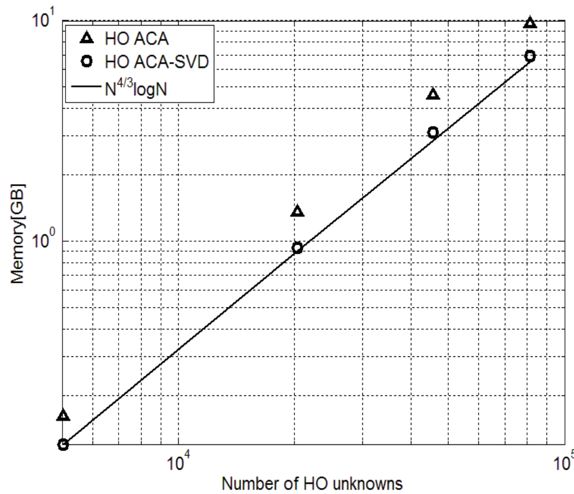
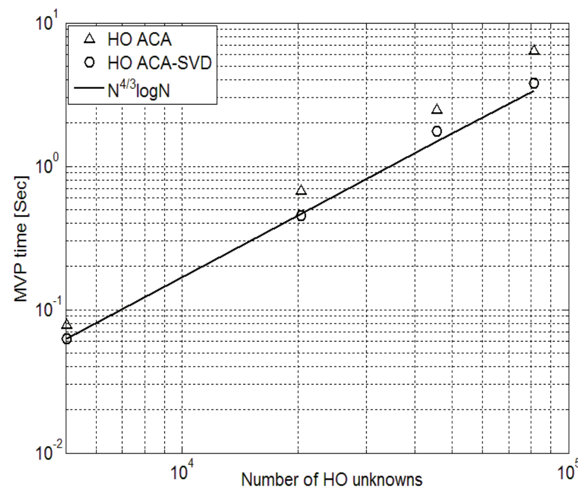


Fig. 4. Bistatic RCS of a PEC cuboid in a two-layer medium for the $\theta^{sca} = 60^\circ$ at 900 MHz.

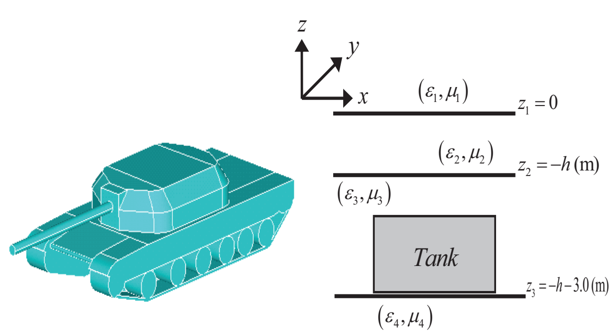


(a)



(b)

Fig. 5. (a) Memory requirement, and (b) CPU time for a matrix-vector operation, as a function of the number of HO unknowns for the cuboid in layered medium.



(a) (b)

Fig. 6. A complex electrically large hard target located in a four-layer medium: (a) the simulation model of tank, and (b) illustration of the tank located in the four-layer medium.

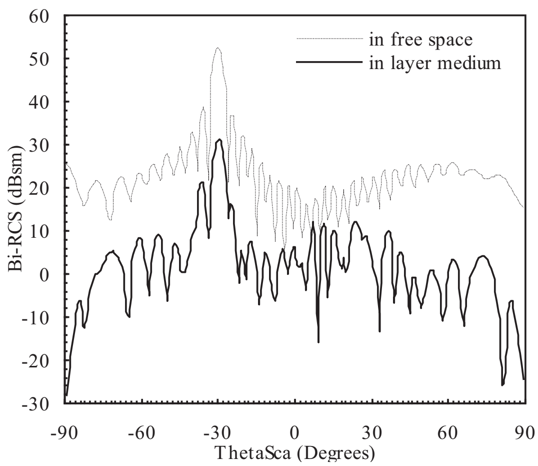


Fig. 7. Bistatic RCS of tank model in layered medium for the $\varphi^{sca} = 0^{\circ}$ at 500 MHz, and the result compared with that in free space.

IV. CONCLUSION

This paper provided an efficient technique for the electromagnetic scattering from the electrically large hard targets located in layered medium, which use the higher order method to discretize the unknown surface current density and the integral equations, and then use the ACA-SVD algorithm to accelerate the higher order MoM solutions. The numerical results demonstrated that the higher order ACA-SVD required much less memory and the CPU time than the low order ACA based algorithm or the higher order MoM solutions.

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REFERENCES

[1] K. Yang and A. E. Yilmaz, "A three-dimensional adaptive integral method for scattering from structures embedded in layered media," *IEEE Trans. Geosci. Remote Sens.*, vol. 50, no. 4, pp. 1130-1139, Apr. 2012.

[2] L. Zhuang, S. Y. He, X. B. Ye, W. D. Hu, and G. Q. Zhu, "The BCGS-FFT method combined with an improved discrete complex image method for EM scattering from electrically large objects in multilayered media," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 3, pp. 1180-1185, Mar. 2010.

[3] E. Jorgensen, J. L. Volakis, P. Meincke, and O. Breinbjerg, "Higher order hierarchical discretization scheme for surface integral equations for layered media," *IEEE Trans. Geosci. Remote Sens.*, vol. 42, no. 4, pp. 764-772, Apr. 2004.

[4] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propag.*, vol. 30, no. 5, pp. 409-418, May 1982.

[5] A. W. Glisson and D. R. Wilton, "Simple and efficient numerical methods for problems of electromagnetic radiation and scattering from surfaces," *IEEE Trans. Antennas Propag.*, vol. 28, pp. 593-602, Sep. 1980.

[6] B. M. Notaros, "Higher order frequency-domain computational electromagnetics," *IEEE Trans. Antennas Propag.*, vol. 56, no. 8, pp. 2251-2276, Aug. 2008.

[7] R. D. Graglia, D. R. Wilton, and A. F. Peterson, "Higher order interpolatory vector bases for computational electromagnetics," *IEEE Trans. Antennas Propag.*, vol. 45, no. 3, pp. 329-342, Mar. 1997.

[8] E. Jorgensen, J. L. Volakis, P. Meincke, and O. Breinbjerg, "Higher order hierarchical Legendre basis functions for electromagnetic modeling," *IEEE Trans. Antennas Propag.*, vol. 52, no. 11, pp. 2985-2995, Nov. 2004.

[9] L. P. Zha, Y. Q. Hu, and T. Su, "Efficient surface integral equation using hierarchical vector bases for complex EM scattering problems," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 952-957, Feb. 2012.

[10] R. Coifman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: a pedestrian prescription," *IEEE Trans. Antennas Propag. Mag.*, vol. 35, no. 3, pp. 7-12, Jun. 1993.

[11] J. M. Song and W. C. Chew, "Multilevel fast multipole algorithm for solving combined field integral equation of electromagnetic scattering," *Microw. Opt. Tech. Lett.*, vol. 10, no. 1, pp. 14-19, Sep. 1995.

[12] S. Velamparambil. W. C. Chew, and J. Song, "10 million unknowns: is it that big?," *IEEE Trans. Antennas Propag.*, vol. 45, no. 2, pp. 43-58, Apr. 2003.

[13] H. P. Zhao, J. Hu, and Z. P. Nie, "Parallelization of MLFMA with composite load partition criteria and asynchronous communication," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 25, no. 2, pp. 167-173, Feb. 2010.

[14] O. Ergul and L. Gurel, "Accurate solutions of extremely large integral equation problems in computational electromagnetics," *IEEE Procedings.*, vol. 101, no. 2, pp. 342-349, Feb. 2013.

[15] K. Zhao, M. N. Vouvakis, and J. F. Lee, "The adaptive cross approximation algorithm for accelerated method of moments computations of EMC problems," *IEEE Trans. Electromang. Compat.*, vol. 47, no. 4, pp. 763-773, Nov. 2005.

[16] M. Bebendorf and S. Kunis, "Recompression techniques for adaptive cross approximation," *J. Integ. Equat. Appl*., vol. 21, no. 3, pp. 331-357, 2009.

[17] T. Su, Z. W. Yang, M. M. Li, and R. S. Chen, "Electrically large 3-D PEC objects buried in a lossy stratified medium," in *Microwave and Millimeter Wave Technology, ICMMT*., pp. 1-4, 2012.

[18] K. A. Michalski and D. Zheng, "Electromagnetic scattering and radiation by surfaces of arbitrary shape in layered media: I. theory," *IEEE Trans. Antennas Propag*., vol. 38, no. 3, pp. 335-344, Mar. 1990.

[19] H. Y. Chao, J. S. Zhao, and W. C. Chew, "Application of curvilinear basis functions and MLFMA for radiation and scattering problems involving curved PEC structures," *IEEE Trans. Antennas Propag*., vol. 51, no. 2, pp. 331-336, 2003.

[20] Z. N. Jiang, R. S. Chen, Z. H. Fan, and M. M. Zhu, "Novel postcompression technique in the matrix decomposition algorithm for the analysis of electromagnetic problems," *Radio Sci*., 47, RS 2003, doi:10.1029/2011RS004921, 2012.

[21] Z. N. Jiang, Z. Fan, D. Ding, R. Chen, and K. W. Leung, "Preconditioned MDA-SVD-MLFMA for analysis of multi-scale problems," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 25, no. 11, pp. 914-925, 2010.

[22] X. Q. Hu, M. Chen, D. Z. Ding, and R. S. Chen, "A novel flexible GMRES with deflated restarted for efficient solution of electromagnetic scattering," *Microwave and Optical Technology Letters*, vol. 53, no. 6, pp. 1360-1364, June 2011.

[23] B. Hu, *Fast Inhomogeneous Plane Wave Algorithm for Electromagnetic Scattering Problems*, Ph.D. dissertation, Dept. Elect. Comput. Eng., Univ. Illinois at Urbana-Champaign, Champaign, IL, 2001.

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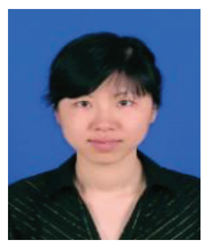

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