High Reflector by One-dimensional Quasi-Periodic Thue-Morse Multilayered Band Gap Structure at Ultra High Frequency Band

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Abstract **─** The quasi-periodic Thue-Morse (T-M) multilayered band gap structures are studied at ultra high frequency band (UHF) by using a theoretical model based on matrix method (MM) for any incidence simulator. It is demonstrated that reflection spectrum in this frequency domain cover a great part of UHF band which contains the Global System for Mobile communication (GSM) bands, where the propagation of electromagnetic waves is forbidden in this region for all incident angle and all polarisations. The study of a deformed stack which is constructed according to the quasi-periodic sequences so that the coordinates y of the deformed object was determined through the coordinates x of the Thue-Morse stack in accordance with the following rule $y=x^{k+1}$, where k: the coefficient defining the deformation degree. Consequently, an omni-directional high reflector at UHF band covering the GSM band is established.

Keywords: Photonic crystals, Thue-Morse sequence, and quasi-periodic multilayer system.

I. INTRODUCTION

Photonic crystals (PC) represent an artificially fabricated system and it is characterized by a periodic dielectric medium where the propagation of an electromagnetic wave is prevented in certain frequency bands independently of the direction of the incident wave and of its polarization.

It has potential applications in many such technological areas as waveguides with photonic crystals, substrates for antennas in microwaves, filters or perfect mirrors and in the development of efficient semiconductor light emitters [1]. In addition, it can also control and manipulate the propagation of electromagnetic waves [2-4].

The simplest form of a photonic crystal is the one-dimensional (1D) periodic structure. It consists of a stack of alternating layers having a low and high refractive indices, whose thicknesses satisfy the Bragg condition: $n_L d_L = n_H d_H = \lambda_0/4$ where λ_0 is the reference wavelength**.** It is known as Bragg Mirror [1, 5, 6].

Quasi-periodic multilayer systems can be considered as suitable models to describe the transition from perfect periodic structures to random structures [7, 8]. Built according to determinist order quasi-periodic multilayer system, the spectra of this multilayer stacks show very interesting physical properties like the existence of a photonic band gap (PBG) for different frequency regions. The Thue-Morse sequence is one of the well-known examples in 1D aperiodic structure [9].

Additionally, some properties of Thue-Morse quasi-crystals have been studied in different domains [10-12], such as the elastic waves in quasi-regular perpendicular polarisation structures, the localization of light waves, which leads to the appearance of photonic band gaps and the propagation of light in Thue-Morse dielectric multilayer systems [13-15].

Also Negro [16] established large omni-directional photonic band gap of 32 layers from Thue-Morse sequence using Si/SiO_2 material and explained its physical origin. He showed the remarkable scaling properties of the transmission spectra both with wavelet decomposition and Fourier Transform Analysis.

Furthermore, by using the tin sulfide–silica material system, Deopura [17] has developed and determined the formation of a broadband visible reflector with an omnidirectional range greater than 10%. Thus, he calculated and measured the reflectance spectra as a function of wavelength for perpendicular (P) and serial (S) polarisations at different incident angles and he established an omnidirectional reflectivity band from 400 to 780 nm.

The deformation of crystals was introduced by applying a power law so that the coordinates y which represent the transformed object, were determined using the coordinates x of the initial object in accordance with the following rule: $y = x^{1+k}$, with k: the coefficient defining the deformation degree.

The initial phase thickness when we apply the y function is: $\varphi = \frac{2\pi}{\lambda} nd\cos\theta$, which takes the following form:

$$\varphi_j = \frac{2\pi}{\lambda} x_0 (j^{k+1} - (j-1)^{k+1}) \cos\theta_j \quad \text{for } j \geq 1.$$

With j designating the j^th layer and $x_0 = \frac{\lambda_0}{4}$ is the thickness of each layer of the periodic structure with λ_0 being the reference wavelength.

For the deformed system, the thickness of each layer becomes variable and depends on the j^{th} layer and the deformation degree k. So, the optical thickness of each layer after deformation by the y function takes the following form: $x'_{0j} = x_0(j^{k+1} - (j-1)^{k+1})$ for $j \geq 1$. It is clear that for a given value of *k*, the quasi-periodic system becomes deformed and the thickness of each layer increases with k increasing. Figure 1 shows, for example, the principle of introducing deformation into the Thue--Morse structure.

Also, the properties of one-dimensional periodic and quasi-periodic photonic crystals with a defect layer have been investigated by Abdel-Rahman et al [18]. In this case, he studied the effect of position, thickness and index of refraction of a defect layer on the transmission spectrum and the defect mode of periodic photonic structure (PPS) as well as Fibonacci quasi-periodic photonic structures.

So in this work we introduce a deformation to the multilayer stack. Deformed stack, which is constructed according to the quasi-periodic generalized Thue-Morse sequence, has been reported in frequency range of the ultra high frequency band. Reflectance spectra reach 100% for the incident angle [0, π/2] and all polarisations. So, the transmission is forbidden in this frequency range.

We have shown that the high frequency term in the spectra depends on both order of system and degree of deformation k.

This paper is organized as follows: in the second part, we give the formulation of Matrix Method (MM), including the reflectance spectrum in both S and P polarisations. The models of quasi-periodic generalised Thue-Morse sequence are presented in section III.

The reflectance spectra of this deformed quasi-periodic multilayer structure are discussed in Section IV, it is shown that a high reflector covers the part of frequency in ultra high frequency band including the GSM band width.

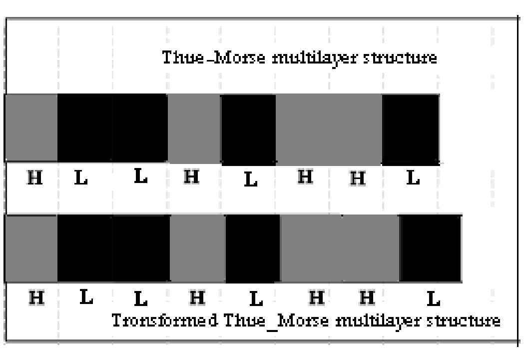

Fig. 1. Transformation of a perfect Thue-Morse multilayer structure into an asymmetric one, for example for h = 0.1.

II. MATHEMATICAL THEORY

We employ the MM, including components of the refractive index, to extract transmission, reflection and consider their sensitivity to material and geometrical variation. It can solve the problem of the photonic band structures and the scattering (transmission and reflection) spectra. For stratified layers within m layer, the amplitudes of the electric fields of incident wave E_0^+, reflected wave E_0^- and transmitted wave E_{m+1}^+ after m layers can be related via the following matrix [19],

$$\begin{pmatrix} E_0^+ \\ E_0^- \end{pmatrix} = \frac{C_1 C_2 C_3 \ldots C_{m+1}}{t_1 t_2 t_3 \ldots t_{m+1}} \begin{pmatrix} E_{m+1}^+ \\ E_{m+1}^- \end{pmatrix}. \quad (1)$$

The C_j (propagation matrix) for the j^{th} sequence can be written,

$$C_j = \begin{pmatrix} \exp(i\phi_{j-1}) & r_j \exp(-i\phi_{j-1}) \\ r_j \exp(i\phi_{j-1}) & \exp(-i\phi_{j-1}) \end{pmatrix} \quad (2)$$

ϕ_{j-1} indicate the phase shift of the wave between $(j-1)^{th}$ and j^{th} boundaries and can be obtained by,

$$\phi_0 = 0 \quad (3)$$

$$\phi_{j-1} = \frac{2\pi}{\lambda} \hat{n}_{j-1} d_{j-1} \cos\theta_{j-1} \quad . \quad (4)$$

The asymmetry was introduced by applying the power lower ,so that the coordinate y which represent the transformed object were determined using the coordinate x of the initial object in accordance with the following rule: $y=x^{k+1}$ here k is the coefficient defining the asymmetry degree in to the Thue-Morse multilayer structure. The initial phase thicknesses when we apply the y function take the following form for $j \geq 1$,

$$\phi_j = \frac{2\pi}{\lambda} \hat{n}_j d_j \cos\theta_j (j^{k+1} - (j-1)^{k+1}) \quad . \quad (5)$$

The Fresnel coefficients t_j and r_j can be expressed as follows by using the complex refractive index $\hat{n}_j$ and the complex refractive angle θ_j .

For parallel P- polarization,

$$r_{jp} = \frac{\hat{n}_{j-1}\cos\theta_j - \hat{n}_j \cos\theta_{j-1}}{\hat{n}_{j-1}\cos\theta_j + \hat{n}_j \cos\theta_{j-1}} \quad (6)$$

$$t_{jp} = \frac{2\hat{n}_{j-1}\cos\theta_{j-1}}{\hat{n}_{j-1}\cos\theta_j + \hat{n}_j\cos\theta_{j-1}}. \quad (7)$$

Moreover, for perpendicular S- polarization,

$$r_{js} = \frac{\hat{n}_{j-1}\cos\theta_{j-1} - \hat{n}_j\cos\theta_j}{\hat{n}_{j-1}\cos\theta_{j-1} + \hat{n}_j\cos\theta_j} \quad (8)$$

$$t_{js} = 2\frac{\hat{n}_{j-1}\cos\theta_{j-1}}{\hat{n}_{j-1}\cos\theta_{j-1} + \hat{n}_j\cos\theta_j}. \quad (9)$$

For both polarisations s and p the transmittance energy T are reduced as,

$$T_S = \mathrm{Re}\left(\frac{\hat{n}_{m+1}\cos\theta_{m+1}}{\hat{n}_0\cos\theta_0}\right)\left|t_S\right|^2 \quad (10)$$

$$T_P = \mathrm{Re}\left(\frac{\hat{n}_{m+1}\cos\theta_{m+1}}{\hat{n}_0\cos\theta_0}\right)\left|t_P\right|^2, \quad (11)$$

Re indicates the real part.

III. GENERALIZED THUE-MORSE MULTILAYER STRUCTURE

One-dimensional quasi-periodic Thue-Morse sequences are multilayer structures consisting of two different materials. They contain two building blocks H and L and can be produced by repeating application of the substitution rules $H \rightarrow HL$ and $L \rightarrow LH$, where H denotes the material with the higher refractive index, and L denotes the material with the lower refractive index. For example, the first few generations S_j of Thue-Morse sequence [20] are as follows: $S_0 = \{H\}$, $S_1 = \{HL\}$, $S_2 = \{HLLH\}$, $S_3 = \{HLLHLHHL\}$ etc.. whereas, the generalized Thue-Morse multilayer is recursively constructed as: $S_{k+1} = (S_k)^n(\overline{S_k})^m$ with $S_1 = H$ and $\overline{S_1} = L$ and arranged according to an inflation rule $\sigma_{T\text{-}M}$: $H \rightarrow H^mL^n$, $L \rightarrow L^mH^n$ [21]. Based on the characteristics of the construction of Thue-Morse sequences, Fig. 2 shows one dimensional generalised Thue-Morse class quasi-periodic multilayer stacks for 3rd generation. According to Thue-Morse rule, there are 16 layers in this structure. (Note: in the all of this work we have chosen m=n=1 i.e., Thue-Morse sequence).

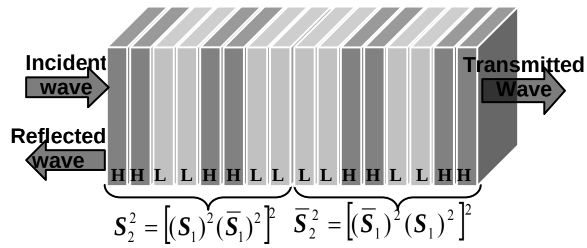

Fig. 2. Schematic representation showing the geometry of the 3rd generation of generalized Thue-Morse quasi-periodic multilayer system for m=n=2.

IV. RESULTS AND DISCUSSIONS

In the following numerical investigation, we choose air (L) and Roger (H) as two elementary layers, with refractive indices n_L=1 and n_H=3, respectively. The thicknesses d_L =3 mm and d_H = 1 mm of the two materials has been chosen to satisfy the Bragg condition: $n_Ld_L = n_Hd_H = \lambda_0/4$ where λ_0=12 mm is the reference wavelength.

We use the matrix method to extract the transmission coefficients in the ultra high frequency spectral range which correspond to 0.3GHz-3GHz.We show that the corresponding reflection coefficients exhibit interesting properties. We assume that the front and the back media have refractive index n_0=1 (index of air). In Fig. 3, we found that the width of forbidden gap [λ_{Long}-λ_{Short}] is sensitive to deformation degree k. We notice, for k=0.05, a high width of band which covers the spectral domain corresponding to ultra high frequency band (0.3<f<3GHz). Therefore, the PBG covers only the GSM band width included in the studied wavelength range: [285 -350] mm.

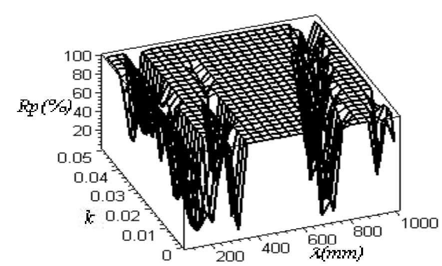

Fig. 3. Reflectance spectra (Rp(%) in mode P) through the 8th level Thue-Morse as function of wavelength and deformation degree k.

The reflectance spectra at 6th order of deformed quasi-periodic Thue-More as functions of wavelength and incident angle θ are shown in Fig. 4. We noticed in this case two basics pseudo photonic band gap. The first forbidden gap is included in GSM band whereas the second band is narrower and covers a greater parts of the wavelength range. So, particularly we interested in the first region which contains a GSM band. We propose a high reflector in this frequency range where this sample structure, built according the Thue-Morse sequence, inhibits the propagation of waves but there exist peaks of transmission in this frequency range for some value of incident angles. Consequently, a partial photonic band gap is established for the 6^{th} level of Thue-Morse sequence. This result is possible via sample periodic structure, but we can't create a transmission peak inside the basic photonic band gap and generate author pseudo band gap outside the original photonic band gap.

At normal incidence, we note a similarity of reflectance spectra for both polarisations. In fact, for $\theta = 0$, the expression of Rs is equal to that of Rp in equation 2. The origin of the fundamental Thue-Morse band gaps can be attributed to local correlations in the form of periodic strings with the corresponding frequency and the layer distributions.

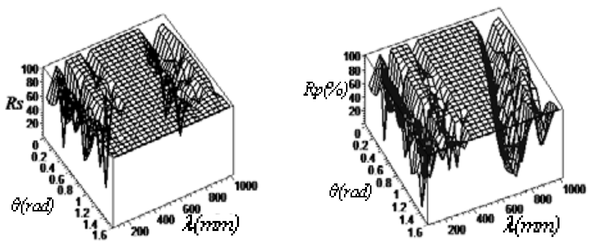

Fig. 4. The reflection spectra through the 6^{th} order of Thue-Morse photonic structures in both modes polarizations (S and P).

It has been shown that multilayer Thue-Morse system with alternating layers do not exhibit a complete photonic band gap where the grey area covers a frequency range for $\theta < 1$ radium (Fig. 5). However, we note another photonic band gap for $\theta > 1$ radium. But their photonic band gap covers a wavelength range between 400 mm and 700 mm where its Fourier spectrum does not exhibit an omni-directional band gap.

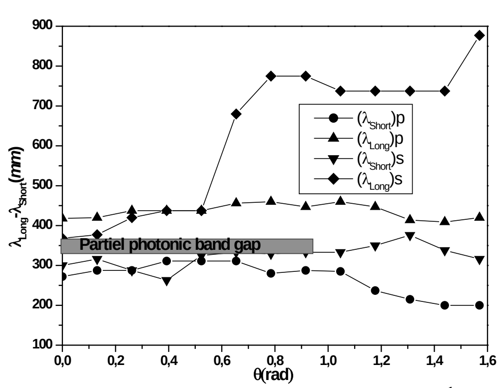

Fig. 5. Variation of limit wavelength of 6^{th} of Thue-Morse sequence in both polarisation S and P as a function of incident angle, with deformation degree (k =0.05).

A. Effect of Varying the Order of Thue-Morse with Constant Deformation

The reflection band for the S mode is wider than that for the P mode. Band gaps are shown for this pair material (air/Roger). Increasing the order of T-M sequence, the width of PBG enhances and narrows transmission peaks increase for two spectra Rp and Rs. It is clear that a PBG depends at Thue-Morse orders. So, from the reflectivity spectrum of S_6 in Fig. 4, the photonic band gap is interrupted for $\theta = 1$ radium and we can noted in this order of Thue-Morse class do not exhibit any omni-directional high reflection Band.

Moreover, with the increase of the order of Thue-Morse sequence (Fig. 6), a large PBG appears from the 8^{th} order of one-dimensional Thue-Morse multilayered structure and covers the GSM band included in ultra high frequency band. So, a fractal Thue-Morse omni-directional band depends on its sample order and on the choice of value of deformation which fixes the number and thicknesses of the layer.

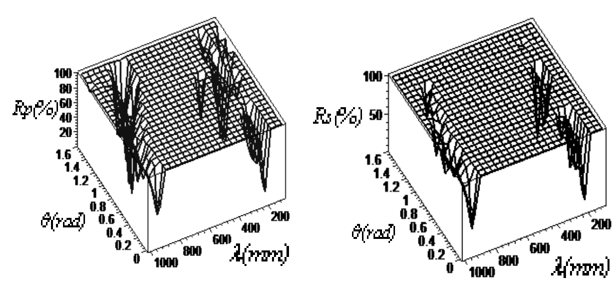

Fig. 6. Reflectance spectra through Thue-Morse multilayer structure as a function of wavelength for TM and TE modes at any incident angles for 8^{th} order of Thue-Morse.

B. Broadening of Omni-directional Photonic Band-Gap

The variation of λ_{long} and λ_{short} for two polarisations S and P marked an omni-directional reflection in a one-dimensional Thue-Morse aperiodic photonic crystal containing the two elements Air and Rogers is determined which broadening versus the order of Thue-Morse sequences where the deformation is fixed to 0.05 (see Fig. 7).

The band gap structure, predicted by the numerical calculations, is clearly identifiable from the 8^{th} Thue-Morse order. Note that this highlight grey area covers all the global systems for mobile communication (GSM) included in ultra high frequency band .which forms a high reflector in this frequency range.

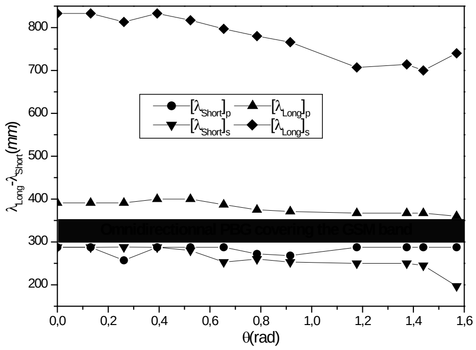

Fig. 7. Variation of limit wavelength of 8th of Thue-Morse sequence in both polarisation S and P as a function of incident angle, with deformation degree (k =0.05).

V. CONCLUSION

Omni-directional reflection through one-dimensional Thue-Morse class quasi-crystal were investigated .We analysed the reflectance spectrum using Matrix method. A forbidden gap has shown in ultra high frequency band which covers the all GSM band width frequency for all incident angles and both polarizations. We also found that a high reflector band gap in ultra high frequency range is generated by all elements of system built according to Thue-Morse sequence.

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