An Efficient ACA Solution for Electromagnetic Scattering from Discrete Body of Revolution

Z. H. Fan, Z. He, and R. S. Chen

Department of Communication Engineering Nanjing University of Science and Technology, Nanjing, 210094, China eerschen@njust.edu.cn

Abstract - Discrete body of revolution (DBoR) enhanced method of moments (MoM) is a specialized technique to analyze electromagnetic scattering from the object with discrete cylindrical periodicity. By exploiting the block circulant property of the impedance matrix of MoM, both filling time and storage requirement for the matrix are reduced. The matrix-vector product can be further accelerated by using the fast Fourier transform (FFT) technique. However, the matrix filling time and memory requirement of DBoR-FFT are the same as those of DBoR-MoM, which are still expensive when the number of unknowns in each sector becomes larger. Meanwhile, the DBoR-FFT scheme works inefficiently for the small number of periodic sectors. In this paper, the adaptive cross approximation (ACA) technique is employed to enhance the DBoR-MoM. Numerical examples are given to demonstrate the efficiency of the proposed method.

Index Terms – Adaptive cross approximation, discrete body of revolution, electromagnetic scattering, method of moments.

I. INTRODUCTION

Method of moments (MoM) is a popular tool to analyze the electromagnetic scattering from the conducting objects [1-3]. For objects with general shape, MoM costs $O(N^3)$ CPU time and $O(N^2)$ memory, where N is the number of unknowns, which prohibits its application to electrically large objects.

There are mainly two types of techniques to conquer this difficulty. On one hand, a series of fast algorithms have been proposed for general geometry [2-4]. Among them, adaptive cross approximation (ACA) algorithm [4] is one of the popular techniques. It makes use of the fact that the approximate rank of submatrix is deficient when the source group and the observation group are sufficiently separated. Hence the submatrix can be computed efficiently by invoking low-rank decomposition technique. On the other hand, specialized codes are developed to save the time and memory cost of the ordinary MoM for the structures bearing the symmetry, uniformity or periodicity. Bodies of revolution [5-7], bodies of translation [8], and periodic frequency selective surface [9] are several well-known structures. Recently, discrete body of revolution (DBoR) based integral equation approach [10-14] is proposed to analyze the structures with discrete cylindrical periodicity.

Many structures encountered in practical application possess discrete cylindrical periodicity, such as windmill, turbine and jet-engine. In the original DBoR schemes [10-11], a matrix equation with multiple right-hand sides has to be solved since the decomposition of incident field are required. A direct solution scheme of DBoR is discussed in [13-14], and it usually requires a parallel out-of-core solver for the electrically large geometries whereas the in-core solution is preferred in most situations. An efficiently iterative DBoR solver, which is free of decomposition of incident field, is proposed in [12]. It exploits the block circulant property of the whole impedance matrix, thus the storage requirement and filling time of the matrix are of order N^2/M , where *M* denotes the number of discrete sectors. The time complexity of one matrix-vector product scales $O(N^2)$ if FFT technique is not adopted, and scales $O[(N^2 \log M)/M]$ if fast Fourier transform (FFT) technique is adopted.

However, the efficiency of DBoR-FFT is still required to be improved since FFT works inefficiently when the number of discrete sectors M is small and the storage requirement and filling time are of $O(N^2/M)$, which is still large. In this paper, a DBoR-ACA scheme is developed which exploits ACA to accelerate the solution of DBoR. Numerical experiments demonstrate that DBoR-ACA is an efficient solution scheme.

The remainder of this paper is organized as follows. In Section II, the theory and the formulations are given. Three numerical experiments are presented in Section III to show the efficiency of the proposed method. Section IV concludes this paper.

II. THEORY AND FORMULATIONS

A. DBoR-MoM and DBoR-FFT

As shown in Fig. 1, consider a DBoR geometry comprised of *M* discrete cylindrically periodic sectors, each sector occupying an angular width of $\Delta \varphi = 2\pi/M$. In the analysis, the mesh is generated for sector S_i and then rotated to obtain the meshes for other sectors S_i , i = 2, 3, ..., M. The meshes must remain conformal on truncated boundary between two neighbor sectors for current continuity and satisfy cylindrically periodical condition to take advantage of DBoR, as discussed in [11]. The surface current density $\mathbf{J}(\mathbf{r})$ is expanded by the RWG basis functions [1] divided into sectors:

$$\boldsymbol{J}(\boldsymbol{r}) = \sum_{m=1}^{M} \sum_{q=1}^{Q} \boldsymbol{I}_{m}^{q} \boldsymbol{f}_{m}^{q}(\boldsymbol{r}) , \qquad (1)$$

where Q denotes the number of unknowns in each sector, and $N = M \times Q$ is the number of total unknowns. I_m^q and f_m^q represent the corresponding expansion coefficient and the RWG function for *q*th basis function in *m*th sector. $r \in S_m$ is position vector. The impedance matrix Z of combined field integral equation (CFIE) is correspondingly partitioned into blocks. As shown in Eq. (2), each block is denoted as Z_{nm} , which represents the interactions between sector S_n and sector S_m , each with the size of $Q \times Q$:

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} & \cdots & \mathbf{Z}_{1M} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} & \cdots & \mathbf{Z}_{2M} \\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} & \cdots & \mathbf{Z}_{3M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{Z}_{M1} & \mathbf{Z}_{M2} & \mathbf{Z}_{M3} & \cdots & \mathbf{Z}_{MM} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \\ \vdots \\ \mathbf{I}_M \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \\ \vdots \\ \mathbf{V}_M \end{bmatrix}.$$
(2)



Fig.1. A geometry with M cylindrical periodic sectors.

Similar to the procedure for electric field integral equation in [12], \mathbf{Z}_{mn} for CFIE can be represented as

 Z_{m-n} since it depends only on the value of m-n. Also, due to the rotational symmetry, it exists $Z_{m-n} = Z_{M+m-n}$. As a result, only Z_i , i=0, 1,..., M-1, are required to be computed and stored. One can also use Q^2 times FFT of *M*-points to compute the matrix-vector product [12] since the impedance matrix is in the block circulant form. In this letter, the scheme without FFT is referred to as DBoR-MoM whereas the scheme with FFT is referred to as DBoR-FFT.

B. DBoR-ACA

DBoR-FFT does not work well when the number of discrete sectors M is small. A scheme of DBoR-ACA is developed by employing the ACA algorithm to compress each block of Z_i . A multilevel spatial partitioning is used to group the RWG functions in each sector. The groups are recorded using octree data at all levels. The touching groups at the finest level are near groups and the others are well-separated groups. The interactions of near groups are computed via DBoR-MoM directly, while the interactions of others are accelerated by the ACA algorithm. Consider two wellseparated groups, one group containing s testing functions residing in sector S_1 , and the other group containing t basis functions residing in sector S_{m} . The interactions between them lead to a submatrix $\mathbf{Z}^{s \times t}$, which is one of the submatrices of block \mathbf{Z}_{M+1-m} . Here, the superscript $s \times t$ denotes the size of the submatrix. Suppose that $\mathbf{Z}^{s \times t}$ is rank-deficient with an effectively approximate rank r. The rank r is usually far smaller than s and t when both s and t are large. By utilizing the ACA algorithm, the matrix $\mathbf{Z}^{s \times t}$ can be approximated as:

$$\mathbf{Z}^{s \times t} \approx \bar{\mathbf{Z}}^{s \times t} = U^{s \times r} V^{r \times t}, \qquad (3)$$

where U^{sxr} and V^{rxt} are two decomposition matrices. The rank *r* is determined adaptively by ACA algorithm to satisfy the following condition:

$$\left\| \mathbf{Z}^{s \times t} - \mathbf{U}^{s \times r} \mathbf{V}^{r \times t} \right\| \leq \tau \left\| \mathbf{Z}^{s \times t} \right\|, \tag{4}$$

where τ denotes the truncated tolerance of the ACA algorithm and is set as 10⁻³ in this paper. The details of the ACA algorithm to fulfill (3) are referred to [4]. DBoR-ACA fills only a fraction of entries for each block Z_i . As shown by numerical observation in [4], both the memory and CPU time requirements of the ACA algorithm scale as $N^{\frac{4}{3}} \log N$ for electrically moderate size problems, while those of MoM scale N^2 . Thus DBoR-ACA can reduce both the memory requirement and simulation time for DBoR-MOM. Table 1 lists a comparison of predictions of the computational complexity for the scheme of DBoR-MoM, DBoR-FFT, and DBoR-ACA, where MVP time denotes the time required to compute matrix-vector product once.

	DBoR		
	MoM	FFT	ACA
Matrix-filling time	N^2 / M	N^2 / M	$N^{\frac{4}{3}}\log N/M$
Storage of matrix	N^2 / M	N^2 / M	$N^{\frac{4}{3}}\log N/M$
MVP time	N^2	$N^2 \log N / M$	$N^{4/3}\log N$

Table 1: Predictions of computational complexity

III. NUMERICAL RESULTS

To demonstrate the efficiency of the DBoR-ACA scheme in comparison with DBoR-MoM and DBoR-FFT, codes are developed for all schemes and numerical examples are presented for typical geometries. In the simulations, the frequency f is 300 MHz unless otherwise specified. The electric field of incident wave is $E^i = \hat{x} \exp(i 2\pi z/\lambda)$, where λ is the wavelength. The discrete rotational axis of DBoR is z axis. CFIE is employed with combination coefficient of 0.5. The resulting matrix equations are iteratively solved by restarted GMRES [16] where the restarted number is set to be 30. The stop criterion for iteration is relative residual norm less than 10⁻³. The bistatic radar cross section (RCS) results are observed at the plane with fixed azimuthal angle $\varphi^s = 0^\circ$ and varied polar angles θ^{s} . All the simulations are carried out in single precision arithmetic on a computer equipped with a 2.83 GHz Intel® Core2 Quad processor, with one core being used.

A. Efficiency test for a conducting ring

The first example is selected to test the performance of different schemes varying with number of sectors. The configuration is a conducting ring with inner radius a=2 m, outer radius b=3 m, and height h=0.1 m, as shown in Fig. 2 (a). To take advantage of DBoR, the mesh has to be changed each time when the number of sectors is increased. Here, the total number of unknowns is kept at a fixed level approximately. Table 2 lists the number of unknowns corresponding to each number of sectors.

Table 2: The number of unknowns and number of sectors for the first example

М	8	16	32	64	80	100	128
Ν	47304	46560	46752	46848	44880	45600	45312

Figure 2 (a) plots the memory requirement for DBoR-MoM and DBoR-ACA. The memory requirement of DBoR-FFT is same as that of DBoR-MoM. It can be observed that the memory requirement of DBoR-MoM is scaled as 1/M, while the DBoR-ACA grows a few larger than O(1/M) as M increases. The reason is that the dividing strategy of the DBoR-ACA group is

controlled by both the octree structure and the sectors. Increasing number of divided sectors brings two burdens which lessening the compressed efficiency of DBoR-ACA against the case when ACA algorithm is utilized for objects of general shape. The first one is that it produces more groups belonging simultaneously to more than one sector, which resulting in more groups with small number of unknowns. The second burden is that the size of the largest group of DBoR-ACA is reduced since the size of each sector is reduced. But even for as many as 128 sectors, the performance of DBoR-ACA has not been reduced by much.

Figure 2 (b) shows the CPU time cost of one MVP. It can be found that the computational time for DBoR-MoM and DBoR-ACA changes slightly as number of sectors increases, whereas the computational time of DBoR-FFT reduces dramatically. The complexity of DBoR-FFT is consistent with $O(N^2 \log N/M)$ for large value of number of sectors M, however the computational time is even greater than that of DBoR-MoM for M < 32. It is because the implementation of FFT with small number of points is not that efficient. In addition, it destroys the CPU cache friendly feature of the submatrices of original DBoR-MoM. It can be observed that a slightly decrease of the CPU time for DBoR-MoM for large number of sectors. This owes to a slightly decreasing of the total number of unknowns as given in Table 2. The slightly increase of the CPU time for DBoR-ACA with large number of sectors is ascribed to the same burdens for the memory requirement. It should be noted that the DBoR-ACA scheme takes more CPU time to perform one MVP than the DBoR-FFT scheme does for large M. For DBoR configuration of practical engineering M is usually not very big, hence, DBoR-ACA is still a faster solver by considering the filling time of the impedance matrix together. Table 3 shows the case for 128 sectors. Here the total time denotes the whole analysis time including the time for pre-processing, matrix filling and solving, and RCS calculating. It can be observed that FFT takes effect in accelerating the DBoR-MoM. However, the total simulation time of DBoR-ACA is still less than that of DBoR-FFT due to saving of matrix filling time.





Fig. 2 Comparisons of the complexity for DBoR-MoM, DBoR-FFT, and DBoR-ACA schemes by increasing the number of divided sectors while keep approximately the total number of unknowns of 46000. (a) Memory, and (b) CPU time for one matrix-vector multiplication.

Table 3: Comparisons of the CPU time and memory requirement for the conducting ring with 128 cylindrical sectors and each with 354 unknowns

	DBoR		
	MoM	FFT	ACA
Memory (MB)	125	125	26
Matrix filling time (s)	72	73	13
Number of iteration	39	39	39
Iteration time (s)	105	19	42
Total time (s)	180	95	58

B. Computation complexity test for a conducting ring

The second example is a conducting ring with inner radius a = 2 m, outer radius b = 4 m, and height h = 0.1 m. This example is to show the complexity of various DBoR schemes for electrically increasing large problems. The ring is discretized with a mesh size of h = 0.05 m and the frequency f is increased from 214.3 MHz to 333.3 MHz. This leads to an increase of the total number of unknowns from 54528 to 129600 as the relation $N \propto f^2$. The ring is modeling with 64 sectors. The size of group box at finest level is 0.2λ in DBoR-ACA scheme. The complexity of memory requirements and CPU time of one MVP for DBoR-MoM and DBoR-ACA are illustrated in Fig. 3 (a) and Fig. 3 (b), respectively. The memory requirement of DBoR-FFT is same as that of DBoR-MoM. It can be observed the practical implementation is consistent with the prediction of the complexity as listed in Table 1. Also both the memory requirement and CPU time cost of DBoR-ACA are less than those of DBoR-FFT when the number of unknowns becomes large in each sector.



Fig. 3. Comparisons of the complexity between DBoR-MoM, DBoR-FFT, and DBoR-ACA algorithms for a conducting ring varying with number of unknowns. (a) Memory, and (b) CPU time for one matrix-vector product.

C. Bistatic RCS for a conducting jet-engine inlet

The last example is a conducting jet-engine inlet as shown in Fig. 4 (a). The configuration has 16 sectors and each sector has 4932 unknowns. The size of group box at finest level of DBoR-ACA algorithm is 0.4 m. This example is to show the efficiency for the small number of sectors of various DBoR schemes. The geometry and dimension of one sector of jet-engine is shown in Fig. 4 (b) and of shell is shown in Fig. 4 (c). For the cylindrical shell, the radius is 2.1 m and the height is 4.0 m for the inner side and the thickness is 0.1 m. The jet-engine is placed at a distance of 0.1 m above the bottom of the shell. The bistatic RCS results are illustrated in Fig. 5 for the various DBoR schemes and fast multipole solver in FEKO®. It can be observed that they are in agreement with each other. Table 4 shows the CPU time and memory requirement for this example. It can be found that DBoR-FFT fails to accelerate DBoR-MoM while DBoR-ACA successes to spend less memory and less CPU time.



Fig. 4. The geometry and mesh of: (a) jet-engine inlet, (b) single sector of jet-engine, and (c) single sector of the shell.



Fig. 5. The bistatic radar cross section of conducting jet-engine inlet.

each sector with 4952 unknowns				
	DBoR			
	MoM	FFT	ACA	
Memory (MB)	3008	3008	396	
Matrix filling time (s)	1530	1532	244	
Number of iteration	344	344	348	
Iteration time (s)	4953	9013	1072	

Table 4: Comparison of the CPU time and memory requirement for the jet-engine inlet with 16 sectors and each sector with 4932 unknowns

IV. CONCLUSION

6490

10552

1321

Total time (s)

The DBoR-MoM has been extended to CFIE for the analysis of electromagnetic scattering from discrete body of revolution in free space. The ACA technique was exploited to accelerate both matrix-filling operation and matrix-vector product of the DBoR-MoM. Numerical examples validate the efficiency and accuracy of the proposed method in comparison with DBoR-MoM and DBoR-FFT. The numerical results suggest that DBoR-FFT fails to accelerate DBoR-MoM for the DBoR with small number of sectors whereas the proposed DBoR-ACA method is appropriate for accelerating the solution of all types of cylindrically periodic geometries. At the end, it is worthwhile to note that a faster scheme can be obtained if sparsified ACA [17] is applied into the DBoR.

ACKNOWLEDGMENT

This work was supported in part by Natural Science Foundation of 61371037, 61431006, 61271076, the Fundamental Research Funds for the Central Universities of No. 30920140111003, No. 30920140121004.

REFERENCES

- S. M. Rao, D. R. Wiltion, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propag.*, vol. 30, no. 3, pp. 409-418, 1982.
- [2] W. C. Chew, J.-M. Jin, E. Michielssen, and J. Song, Fast and Efficient Algorithms in Computational Electromagnetics. Boston, MA: Artech House, 2001.
- [3] W. C. Chew and L. J. Jiang, "Overview of largescale computing: The past, the present, and the future," *Proceedings of the IEEE*, vol. 101, no. 2, pp. 227-241, 2013.
- [4] K. Z. Zhao, M. N. Vouvakis, and J.-F. Lee, "The adaptive cross approximation algorithm for accelerated method of moments computations of EMC problems," *IEEE Transactions on Electromagnetic Compatibility*, vol. 47, no. 4, pp. 763-773, 2005.
- [5] J. R. Mautz and R. F. Harrington, "Radiation and scattering from bodies of revolution," *Applied*

Scientific Research, vol. 20, no. 1, pp. 405-435, 1969.

- [6] Z. He, H. H. Zhang, and R. S. Chen, "Parallel marching-on-in-degree solver of time-domain combined field integral equation for bodies of revolution accelerated by MLACA," *IEEE Trans. Antennas Propag.*, vol. 63, no. 8, pp. 3705-3710, 2015.
- [7] Z. He, Z. H. Fan, D. Z. Ding, and R. S. Chen, "Solution of PMCHW integral equation for transient electromagnetic scattering from dielectric body of revolution," *IEEE Trans. Antennas Propag.*, vol. 63, no. 11, pp. 5124-5129, 2015.
- [8] L. Medgyesi-Mitschang and J. Putnam, "Scattering from finite bodies of translation: Plates, curved surfaces, and noncircular cylinders," *IEEE Trans. Antennas Propag.*, vol. 31, no. 6, pp. 847-852, 1983.
- [9] B. Munk, Frequency Selective Surfaces: Theory and Design. John Wiley, 2000.
- [10] H. T. Anastassiu, J. L. Volakis, and D. S. Fili, "Integral equation modeling of cylindrically periodic scatterers in the interior of a cylindrical waveguide," *IEEE Transactions on Microwave Theory and Techniques*, vol. 46, no. 11, pp. 1713-1720, 1998.
- [11] M. A. Carr, J. L. Volakis, and D. C. Ross, "Acceleration of moment method solutions for discrete bodies of revolution in free space," *IEEE Antennas and Propagation Society International Symposium*, vol. 4, pp. 2286-2289, 2000.
- [12] M. A. Carr, J. L. Volakis, and D. C. Ross, "Acceleration of free-space discrete body of revolution codes by exploiting circulant submatrices," *IEEE Trans. Antennas Propag.*, vol. 50, no. 9, pp. 1319-1322, 2002.
- [13] P. Soudais, P. Leca, J. Simon, and T. Volpert, "Computation of the scattering from inhomogeneous objects with a discrete rotational symmetry and a nonsymmetric part," *IEEE Trans. Antennas Propag.*, vol. 50, no. 2, pp. 168-174, 2002.
- [14] H. T. Anastassiu, N. L. Tsitsas, and P. J. Papakanellos, "Electromagnetic scattering and radiation by discrete bodies of revolution," URSI Int. Symp. on Electromagnetic Theory, Berlin, Germany, pp. 657-659, 2010.
- [15] J. M. Song and W. C. Chew, "Multilevel fastmultipole algorithm for solving combined field integral equations of electromagnetic scattering," *Microwave Opt. Tech. Lett.*, vol. 10, no. 1, pp. 14-19, 1995.
- [16] Y. Saad and M. H. Schultz, "GMRES: A generalized minimal residual algorithm for solving nonsymmetric linear systems," SIAM

Journal on Scientific & Statistical Computing, vol. 7, no. 3, pp. 856-869, 1986.

[17] A. Heldring, J. M. Tamayo, C. Simon, E. Ubeda, and J. M. Rius, "Sparsified adaptive cross approximation algorithm for accelerated method of moments computations," *IEEE Trans. Antennas Propag*, vol. 61, no. 1, pp. 240-246, 2013.



Zhenhong Fan was born in Jiangsu, China, in 1978. He received the M.Sc. and Ph.D. degrees in Electromagnetic Field and Microwave Technique from Nanjing University of Science and Technology (NJUST), Nanjing, China, in 2003 and 2007,

respectively.

During 2006, he was with the Center of Wireless Communication in the City University of Hong Kong, Kowloon, as a Research Assistant. He is currently an Associate Professor with the Electronic Engineering of NJUST. He is the author or co-author of over 20 technical papers. His current research interests include computational electromagnetics, electromagnetic scattering and radiation.



Zi He received the B.Sc. degree in Electronic Information Engineering from the School of Electrical Engineering and Optical Technique, Nanjing University of Science and Technology, Nanjing, China, in 2011.

She is currently working towards the Ph.D. degree in Electromagnetic Fields and Microwave Technology at the School of Electrical Engineering and Optical Technique, Nanjing University of Science and Technology. Her research interests include antenna, RF-integrated circuits, and computational electromagnetics.



Rushan Chen (M'01) was born in Jiangsu, China. He received the B.Sc. and M.Sc. degrees from the Department of Radio Engineering, Southeast University, China, in 1987 and 1990, respectively, and the Ph.D. degree from the Department of Electronic

Engineering, City University of Hong Kong, in 2001.

He joined the Department of Electrical Engineering, Nanjing University of Science and

Technology (NJUST), China, where he became a Teaching Assistant in 1990 and a Lecturer in 1992. Since September 1996, he has been a Visiting Scholar with the Department of Electronic Engineering, City University of Hong Kong, first as Research Associate, then as a Senior Research Associate in July 1997, a Research Fellow in April 1998, and a Senior Research Fellow in 1999. From June to September 1999, he was also a Visiting Scholar at Montreal University, Canada. In September 1999, he was promoted to Full Professor and Associate Director of the Microwave and Communication Research Center in NJUST, and in 2007, he was appointed as the Head of the Department of Communication Engineering, NJUST. He was appointed as the Dean in the School of Communication and Information Engineering, Nanjing Post and Communications University in 2009. And in 2011 he was appointed as Vice Dean of the School of Electrical Engineering and Optical Technique, NJUST. Currently, he is a Principal Investigator of more than 10 national projects. His research interests mainly include computational electromagnetics, microwave integrated circuit and nonlinear theory, smart antenna in communications and radar engineering, microwave material and measurement, RF-integrated circuits, etc. He has authored or co-authored more than 260 papers, including over 180 papers in international journals.

Chen is an Expert enjoying the special government allowance, Member of Electronic Science and Technology Group, Fellow of the Chinese Institute of Electronics (CIE), Vice-Presidents of Microwave Society of CIE and IEEE MTT/APS/EMC Nanjing Chapter and an Associate Editor for the International Journal of Electronics. He was also the recipient of the Foundation for China Distinguished Young Investigators presented by the China NSF, a Cheung Kong Scholar of the China Ministry of Education, New Century Billion Talents Award. Besides, he received several Best Paper Awards from the National and International Conferences and Organizations. He serves as the Reviewer for many technical journals, such as the IEEE Transactions on Antennas and Propagation, the IEEE Transactions on Microwave Theory and Techniques, Chinese Physics, etc.