Hybrid Sparse Reconstruction-Method of Moments for Diagnosis of Wire Antenna Arrays

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Abstract **—** In this work, sparse reconstruction is hybridized with method of moments (MoM) for diagnosis of wire antenna arrays. In the hybrid method, excitation voltages of array under diagnosis are related to near field observations through integral equations (IEs), and IEs are transformed into matrix equations by MoM. Sparse reconstruction is then used to solve excitation voltages of array under diagnosis. Finally, locations of failing elements are found by the solved excitation voltages. The hybrid method models mutual coupling effect by MoM, and it achieves higher diagnosis reliability than sparse reconstruction without mutual coupling consideration. Simulation results are presented to show the validity and advantages of the hybrid method.

Index Terms **—** Array diagnosis, integral equation, method of moments, sparse reconstruction.

I. INTRODUCTION

Large scale antenna arrays are widely used in radar systems and radio telescopes. If some elements of an array are not working properly, the array radiation pattern can be significantly affected. In order to repair a failing array, it is essential to develop fast and reliable array diagnosis methods. Recently, array diagnosis based on sparse reconstruction attracted much interest. The key advantage of sparse reconstruction is the reduction of required observations. This was first demonstrated by numerical simulations using planar arrays [1]. Theoretical proof was later derived for uniform linear arrays [2]. Besides, estimation of diagnosis reliability was developed in [3].

In practice, diagnosis reliability is affected by various factors, including element positioning error, measurement noise, mutual coupling between array elements, etc. Effects of element positioning error and Gaussian noise were discussed in [1], and non-Gaussian noise was studied in [4]. Toeplitz property of the mutual coupling matrix was utilized to model the mutual coupling of uniform linear array [5]. However, mutual coupling of arbitrarily shaped arrays has not been considered in existing work.

In order to model the effect of mutual coupling, this work combines sparse reconstruction with method of moments (MoM). Using MoM, a matrix equation linking excitation voltages and near field observations is first derived from integral equations. Excitation voltages are then solved by sparse reconstruction to locate failing elements. The main advantage of the proposed method is the inclusion of mutual coupling effect of arbitrarily shaped arrays, which is shown to significantly improve diagnosis reliability.

It should be noted that there have been MoM-based source reconstruction methods for solving inverse problems [6, 7]. Unknowns in these methods are equivalent currents which are usually not sparse. On the other hand, unknowns in the proposed method are excitation voltages, which can be made sparse and solved by sparse reconstruction. Furthermore, the support vector machine (SVM) based-source identification method was proposed in [8]. The SVM based-source identification requires a training process. It is able to detect the radiation source efficiently once the training process is completed. For the large scale arrays considered in this work, the training process may be tedious.

II. THE PROPOSED METHOD

Consider the wire antenna array shown in Fig. 1. Fields are observed on an observation plane and they are used to reconstruct the excitation voltages.

A. Relating excitation voltages and observed fields

Assuming the wire antenna is thin (the radius of the wire is less than one hundredth of the wavelength), electromagnetic behavior of the array is described using the thin wire integral equation [9]. Using MoM, the thin wire integral equation is discretized and the following matrix equation is derived:

$$\mathbf{Zc}=\mathbf{f}, \tag{1}$$

where $\mathbf{Z}$ is the interaction matrix, $\mathbf{c}$ is an unknown vector constituted by current expansion coefficients, and $\mathbf{f}$ is the excitation vector arising from excitation voltages. The dimension of $\mathbf{Z}$ is $BN_J \times BN_J$, and the dimension of $\mathbf{c}$ and $\mathbf{f}$ is $BN_J \times 1$, where N_J is the number of basis functions

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for one antenna element and B is the total number of antenna elements. Definitions of $\mathbf{Z}$ and $\mathbf{f}$ are given in Equations (4.68) and (4.69) of [9], and they are omitted here. It should be mentioned that the mutual coupling effect is automatically included in the interaction matrix $\mathbf{Z}$.

At the q-th observation point $\vec{r}_q$ (q=1,2,...,Q, where Q is the total number of observations), electric field is calculated using the thin wire integral equation. Substituting current expansion into the thin wire integral equation, the following matrix equation is obtained:

$$\mathbf{E} = \mathbf{Uc}, \tag{2}$$

where $\mathbf{E}$ is a vector constituted by E_q and E_q is electric field at the q-th observation point. $\mathbf{U}$ is a complex-valued matrix arising from point matching of the thin wire integral equation, and its dimension is $Q \times BN_J$. The excitation vector $\mathbf{f}$ is written as:

$$\mathbf{f} = \mathbf{Tv}, \tag{3}$$

where $\mathbf{v}$ is a column vector constituted by V_b (b = 1, 2, ..., B) and V_b is the excitation voltage at the b-th feed port. $\mathbf{T}$ is a $BN_J \times B$ complex-valued matrix defined as:

$$T_{m,b} = \frac{-j}{\omega\mu\delta_b}\int_{P_b} s_m(\vec{r})d\vec{r}\ , \tag{4}$$

where P_b is the line segment where the b-th feed port is located, δ_b is the length of P_b, and s_m is the m-th triangular basis function. Substituting (3) into (1), we have:

$$\mathbf{Zc} = \mathbf{Tv}. \tag{5}$$

From (5) and (2), it is observed that matrix $\mathbf{Z}$ needs inverting in order to obtain an explicit linear relationship between excitation voltages $\mathbf{v}$ and observed fields $\mathbf{E}$. When the size of $\mathbf{Z}$ is large, it is difficult to directly invert $\mathbf{Z}$. In that case, fast integral equation methods should be used to accelerate the computation of $\mathbf{Z}$ and its inversion [10]. In this work, we assume that the inversion of $\mathbf{Z}$ can be computed directly. Equation (5) is then rewritten as:

$$\mathbf{c} = \mathbf{Z}^{-1}\mathbf{Tv}, \tag{6}$$

where $\mathbf{Z}^{-1}$ is the inversion of $\mathbf{Z}$. Substituting (6) into (2), excitation voltages are related to observed fields as:

$$\mathbf{E} = \mathbf{Yv}, \tag{7}$$

with $\mathbf{Y} = \mathbf{UZ}^{-1}\mathbf{T}$.

B. Finding excitation from observation

In order to utilize advantages of sparse solution, this work adopts the differential formulation proposed in [1] for small failing rate. According to the differential formulation [1], the following equation is derived from (7):

$$\mathbf{E}_\delta = \mathbf{Yv}_\delta, \tag{8}$$

where $\mathbf{E}_\delta = \mathbf{E} - \mathbf{E}_{ref}$ and $\mathbf{v}_\delta = \mathbf{v} - \mathbf{v}_{ref}$. $\mathbf{E}_{ref}$ and $\mathbf{v}_{ref}$ are observed fields and voltage sources of the normal array, and they are assumed known in advance. $\mathbf{v}_\delta$ is sparse when the failing rate is small. Therefore, $\mathbf{v}_\delta$ can be estimated using ℓ_1-norm solver. Once $\mathbf{v}_\delta$ is obtained, $\mathbf{v}$ is recovered as $\mathbf{v}_{ref} + \mathbf{v}_\delta$. In practice, the failing rate is unknown. Hence, one may reconstruct the voltage excitation source using both (7) and (8) and choose the one whose radiating fields are closer to the observed fields. Taking Equation (7) as an example, $\mathbf{v}$ is obtained by solving the following optimization problem:

$$\min_{\mathbf{v}}\left(\|\mathbf{v}\|_p\right)^p, \tag{9a}$$

$$\text{subject to } |E_m - \mathbf{Y}_m\mathbf{v}| \le \epsilon,\ \forall m, \tag{9b}$$

where $||\mathbf{v}||_p$ is the ℓ_p-norm of $\mathbf{v}$, E_m is the m-th element of $\mathbf{E}$, $\mathbf{Y}_m$ is the m-th row of $\mathbf{Y}$, and ϵ is the tolerance. When p = 2, ℓ_2-norm solution is obtained, which results in the minimum root mean square error solution. If p = 1, a sparse solution will be derived. When the solution is sparse, ℓ_1-norm solver renders better accuracy than ℓ_2-norm solver. The solution of (9) is detailed in [4], and it is omitted here.

The computational complexity of solving the optimization problem in (9) is the same as the array diagnosis method presented in [4]. Because the mutual coupling effect is considered in the proposed method, the matrix filling requires longer CPU time. In order to save the CPU time, matrix $\mathbf{Y}$ in (8) can be pre-stored and reused.

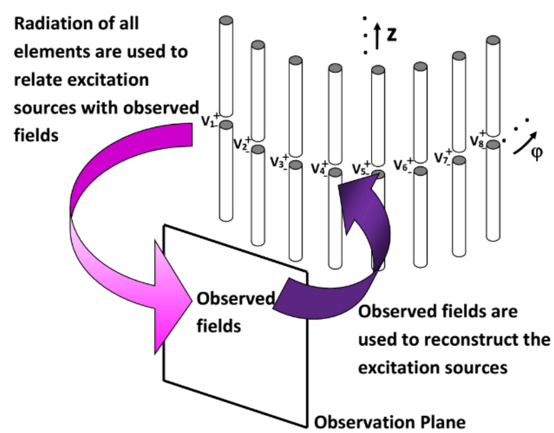

Fig. 1. Diagnosis of a dipole array. The excitation voltage at the m^{th} feed port is denoted by V_m.

III. SIMULATION RESULTS

A. Accuracy study

This section presents simulation results to study the accuracy of our proposed method and to demonstrate the accuracy improvement by considering mutual coupling.

1) Simulation Setup

The array under consideration is a 10 × 10 cylindrical array consisting of half-wavelength dipole antennas. The array elements are distributed on a z-directed cylinder of radius 5λ with $z \in [0, 9.5\lambda]$ and $\phi \in [-0.45, 0.45]$, where λ denotes the wavelength. The elemental spacings in z- and ϕ-directions are 1.0λ and 0.1 radian, respectively.

Radiated fields are observed on a rectangular plane defined as $x = 6\lambda$, $y \in [-2.25\lambda, 2.25\lambda]$, and $z \in [0, 9.5\lambda]$. For validation purpose, radiated near fields of the array are first calculated with uniform voltage excitation of 1 V. Results from our code agree well with those from commercial software FEKO. This validates the calculation of matrices **Z**, **T**, and **U**. Our code is then used to generate observed fields with the voltage excitation following Gaussian distribution. z-component of the electric field is used for failure diagnosis. Furthermore, Gaussian noise is added to the synthetic data in order to simulate the measurement uncertainty in experiment. To enhance the robustness against Gaussian noise, the quadratic loss function is adopted in the unsupervised support vector regression [4]. The relative root mean square error (RMSE) is used to measure the accuracy of the reconstructed source. In order to consider the effect of location of failing elements, RMSE is obtained by 50 independent simulations, each with random locations of failing elements.

2) Accuracy for Sparse Solution

Figure 2 presents RMSE against the number of observations for sparsity of 92% and 96%. The signal to noise ratio (SNR) is fixed to 40 dB. It is seen that RMSE reduces as the number of observations increases, with the reduction rate becoming slow after certain number of observations. This indicates that there is a critical point Q_c, after which adding extra observation data provides insignificant accuracy improvement. Figure 2 shows that the value of Q_c is smaller for the case of 96% solution sparsity. Hence, sparser solution requires smaller Qc. For practical application, Q_c may be determined by gradually adding observation data until the convergence of the solution vector [11]. Figure 2 also compares RMSE with and without mutual coupling consideration. It is seen that the accuracy is improved by one order of magnitude when mutual coupling is considered. Meanwhile, the accuracy improvement is higher for the case of 96% solution sparsity. This is because higher solution sparsity results in a larger number of passive radiating elements, which makes the role of mutual coupling more important.

Figure 3 shows RMSE versus SNR, where the number of observations is fixed to 100. It is seen that RMSE with and without mutual coupling consideration is large when SNR is below 20 dB. However, RMSE is lower when mutual coupling is considered. Meanwhile, RMSE is reduced by one order of magnitude via considering mutual coupling when SNR is larger than 20 dB. Furthermore, RMSE reduction is larger when solution sparsity is 96%. This agrees with the observation from Fig. 2.

3) Accuracy Against Sparsity of the Solution

In the case of Figs. 2 and 3, the solution sparsity is high. It is interesting to see how the accuracy is affected by the solution sparsity. Figure 4 presents RMSE with solution sparsity dropping from 95% to 50%. As the solution sparsity drops, RMSE gradually rises. In the case of SNR=40 dB, the accuracy improvement by considering mutual coupling is one order of magnitude for all solution sparsity. When SNR is 20 dB, RMSE is high for both with and without mutual coupling consideration. Figure 5 shows the rate of correct detection using the detection criterion presented in [4]. For both values of SNR, considering mutual coupling increases the rate of correct detection. When SNR is 40 dB, the proposed method provides 100% rate of correct detection for all solution sparsity. When SNR is 20 dB, the rate of correct detection is 98% for sparsity of 95%. However, as the sparsity decreases, the rate of correct detection drops. For low sparsity, the rate of correct detection may be improved by increasing the number of observations [4].

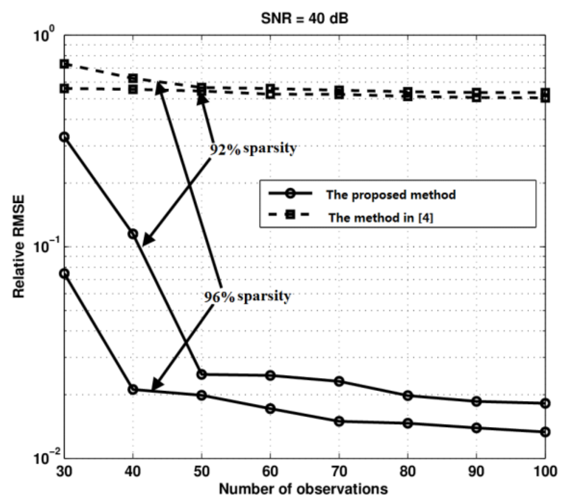

Fig. 2. RMSE versus the number of observations.

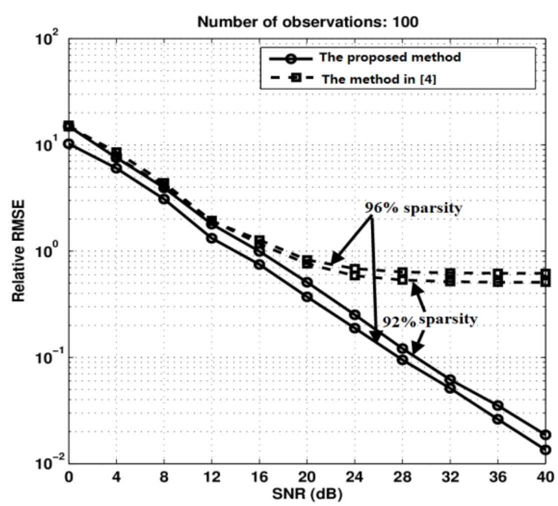

Fig. 3. RMSE versus SNR.

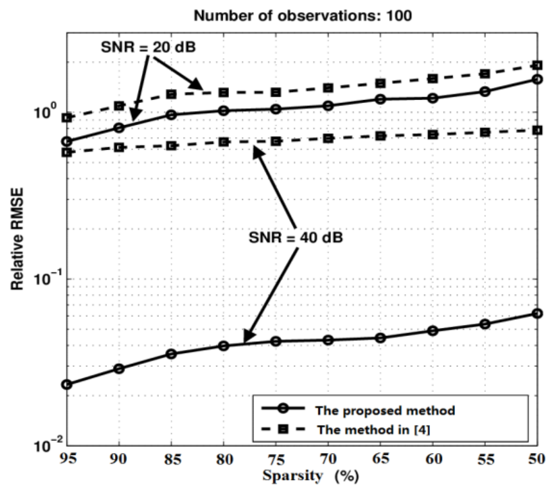

Fig. 4. RMSE versus solution sparsity.

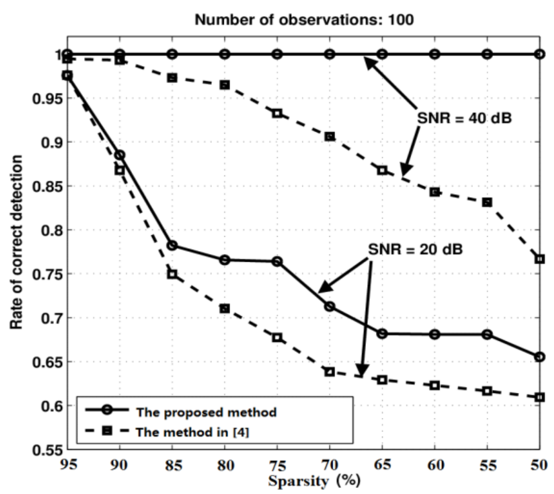

Fig. 5. Rate of correct detection versus solution sparsity.

B. Application to a 40 × 30 cylindrical dipole array

The proposed method is applied to diagnose a 40 × 30 cylindrical array constituted by half-wavelength dipole antennas. The array is located on a z-directed cylinder with radius of 5λ, $\phi \in [-0.975, 0.975]$ and $z \in [0, 21.75\lambda]$. The elemental spacings in ϕ- and z-directions are 0.05 radian and 0.75λ, respectively. The voltage excitation of the array is shown in Fig. 6 (a), with failing elements forming an 'E'-shape. The z-component of radiated electric field is observed on a plane defined as $x = 6\lambda$, $y \in [-5\lambda, 5\lambda]$ and $z \in [-0.25\lambda, 22\lambda]$, with 41 and 31 field points in y- and z-directions, respectively. SNR is set to 20 dB. It is assumed that the failing rate is unknown, and the voltage excitation source is reconstructed using both (7) and (8). The reconstructed voltage source with smaller approximation error is chosen. Figure 6 (b) presents the reconstructed voltage excitation with mutual coupling consideration. It is seen that the failing elements are correctly detected when mutual coupling is considered. Figure 6 (c) shows the reconstructed voltage source without mutual coupling consideration, where locations of failing elements are obscure.

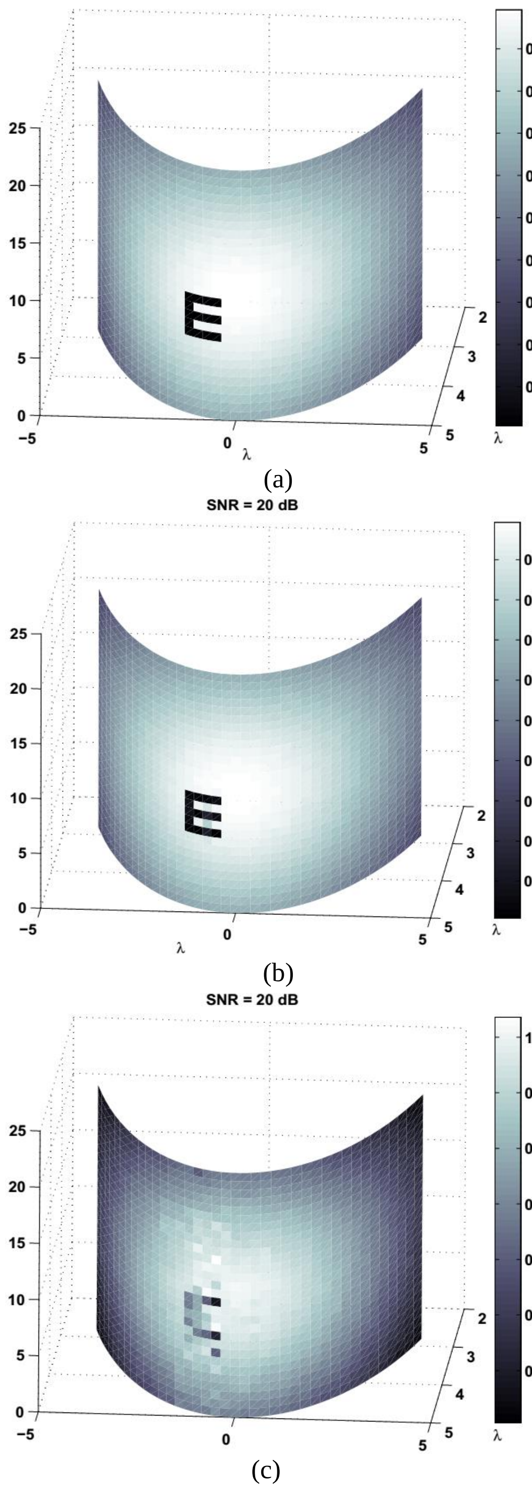

Fig. 6. Excitation source of the large cylindrical dipole array: (a) original, (b) reconstructed by the proposed method, and (c) reconstructed by the method in [4].

IV. CONCLUSION

This paper presented diagnosis of wire antenna arrays by hybrid sparse reconstruction-MoM. Mutual coupling between array elements has been taken into account by MoM. Simulations have been conducted to study the accuracy of the proposed method. It has been observed that the accuracy can be improved by one order of magnitude when taking mutual coupling into consideration. Furthermore, application to a large array has been demonstrated. It has been found that failing elements are correctly located when mutual coupling is considered. On the other hand, ignoring mutual coupling results in obscure locations of failing elements. Although the array elements considered here are half-wavelength dipole antennas, the proposed method can deal with other type of elements because of the flexibility of MoM.

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