Electromagnetic Point Source Reconstruction by Reversed-TLM Method

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Abstract **─** Classical methods for source synthesis are iterative, time consuming, and not always adapted to the desired problem. In this paper, we present a new method of electromagnetic source synthesis based on the time-reversal technique. This approach employs the *reversed*-TLM method and permits the reconstruction of a source distribution, from its electromagnetic far-field radiation. Point-like source reconstruction results show that by using this method, the "classical" half-wavelength resolution limit is overcome.

Index Terms **─** Electromagnetic simulation, inverse problem, source synthesis, TLM method.

I. INTRODUCTION

Traditional methods for radiating electromagnetic (EM) structure design typically begin by defining the architecture of the source distribution. An iterative procedure is then developed to ensure that the design process converges to an optimal solution [1-3]. Even if performance of such techniques in the frequency domain (FD) is good, convergent procedure is not efficient in the time domain (TD). Time reversal (TR) comes as a natural method for EM structure design. This technique was first applied in acoustics [4-6], and more recently in electromagnetics [7-9], answering some important questions, like those related to inverse problem solution non-uniqueness [10], and resolution limitations [11, 12].

Our purpose is to use this TR wave theory and to introduce an innovative TD based algorithm adapted to EM source synthesis. The numerical method that we use is the *reversed*-TLM (transmission-line matrix method), based on symmetrical condensed nodes (SCN) [13].

This paper is basically focused on EM point source reconstruction, using the radiating field outside the region containing the sources. Theoretical concepts as "time reversal mirrors" (TRM) [6] and "time reversal cavity" (TRC) [8] are used. Inverse source problems (ISP) have been already studied, analytically, in FD [14, 15] and in TD [16]. TLM method has been previously used for solving numerous EM problems, which cover various domains of applications. The basics of the time-reversed TLM process have been introduced in [17]. This technique has been applied previously in ISP [18], microwave filter synthesis [19-21], and inverse diffraction applications [22, 23].

In inverse diffraction problems [23], the object to be reconstructed is illuminated by a plane wave excitation. Two direct simulations are performed: a first one with the object and a second one without the object. The EM field history is recorded, in both cases, on the TRC surface. The difference between the outputs of the two simulations, i.e. the diffracted field, is injected in the reversed simulation. In our case, there is no plane wave excitation. Besides, only one direct TLM simulation is performed. Our goal is to find the information to be added at the beginning of the TR approach, in order to find the initial source distribution, by a reversed TLM simulation only. In this way, the knowledge of amplitudes and phases of the radiating field and the knowledge of the excitation would be sufficient to find primary or secondary sources.

Our method is first applied to lumped wide band sources, in the frequency range [26GHz - 34GHz], placed in a lossless, homogeneous, and non-dispersive 3D free-space. Results show that the well-known spatial resolution limitation of the

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reconstruction is solved. A sub-wavelength resolution is achieved.

II. RETRO-PROPAGATION BY REVERSED-TLM METHOD

Theory of wave propagation TR is based on the invariance property of the scalar wave equation (1) under TR transformation, in a lossless space:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},\vec{r}_0,t) = 0, \qquad (1)$$

where $\psi(\vec{r},\vec{r}_0,t)$ is the scalar radiated field, $\vec{r}$ is the distance-vector between the source and the observation point, $\vec{r}_0$ is the source position, t is the time and c is the speed of light.

This equation is a differential one, containing only a second order derivative with respect to time. Therefore, if $\psi(\vec{r},\vec{r}_0,t)$ is a solution of this equation, then $\psi(\vec{r},\vec{r}_0,-t)$ is also a solution [24], [25]. In other words, scalar wave equation remains invariant under TR transformation. This property is valid only for a non-absorbing medium.

It can be shown that Maxwell's equations are also symmetrical under TR [25]. So, if $\vec{E}(\vec{r},t)$ and $\vec{H}(\vec{r},t)$ are solutions of Maxwell's equations, then $\vec{E}(\vec{r},-t)$ and $-\vec{H}(\vec{r},-t)$ are solutions of the same set of equations. Following Huygens's principle, the knowledge of the radiated field on a closed surface surrounding the sources is sufficient to recreate the field inside the entire volume.

We use the 3D *reversed*-TLM method, with SCN nodes, to numerically simulate EM wave retro-propagation in TD. The propagation space is represented by a mesh of interconnected transmission lines [13].

Reversed-TLM simulation is identical to the direct one due to the scattering node matrix uniqueness property ($[S]^{-1}[S] = I$) [17]. Thus, an EM radiation process can be theoretically time-reversed without any change. The unique difference is that initial conditions change.

III. RECONSTRUCTION PROCEDURE

Our new algorithm based on 3D wave retro-propagation, permits the reconstruction of an unknown source distribution from its radiated field. So, despite the fact that the solution to inverse problems is not unique, we are proving the feasibility of this method under some circumstances. Thus, some *a priori* information about the solution of the initial inverse problem is added. The knowledge of the transmitted waveform transforms the initial ill-posed problem in a well-posed one, with a unique solution.

We developed a *two-step* method to determine the position and the dimensions of the sources from measured- or theoretically-computed radiated-field values. The two steps of the method are: the coarse reconstruction step and the resolution improvement one. We exploit this method as follows.

A. Coarse reconstruction step

From the sampled values of the desired far-field radiation, we reconstruct the excitation to apply in each point of the cubical TRC external surface. The six TRC faces are called TRMs (Fig. 1a). In order to create the proper wave re-composition during the TR process, the excitation of the TLM nodes, on the six TRMs, needs delay compensation. Hence, for a proper reconstruction of the excitation, we also need the delay information at each point of the TRMs. Reconstructed signals are injected in the TLM network, in each point of the TRM. Reversed-TLM method is then applied.

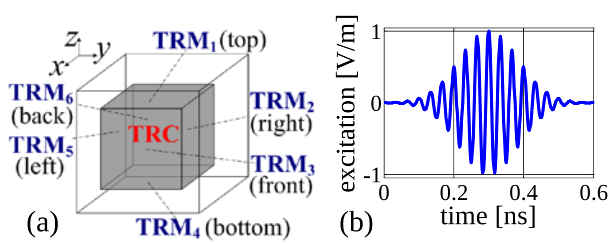

Fig. 1. (a) TRC 3D; (b) excitation signal: *f*(*t*).

Considering the new established boundary conditions on the TRMs, a time-reversed field (ψ_{TR}) retro-propagates in all the volume [4]. This field satisfies the Helmholtz equation without sources (2).

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi_{TR}(\vec{r},t) = 0. \qquad (2)$$

As a consequence, ψ_{TR} has the following form:

$$\psi_{TR}(\vec{r},\vec{r}_0,t) = \\ = G(\vec{r},\vec{r}_0,-t) * f(-t) - G(\vec{r},\vec{r}_0,t) * f(-t), \qquad (3)$$

where $G(\vec{r},\vec{r}_0,t)$ is the Green's function, *f*(*t*) is the initial temporal excitation and "$*$" is the temporal convolution operator.

Thus, the TR operation generates a convergent wave focusing on the initial source position (Fig. 2a) and a divergent wave, which appears after the collapse (Fig. 2c). So, in the proximity of the initial source position there is a superposition of these two waves (Fig. 2b).

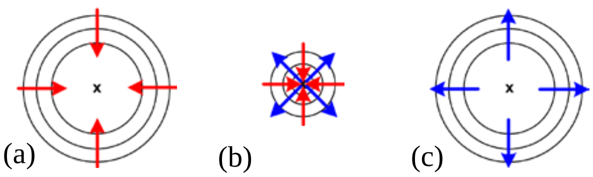

Fig. 2. Time-reversed waves' focalization; (a) convergent wave; (b) interference between convergent and divergent wave in the proximity of the source; (c) divergent wave.

Therefore, if we apply the *reversed*-TLM method based on this approach, TR field does not focus perfectly on the initial source position. Besides, even if both waves have a singularity in $\vec{r}_0$, the TR field has a finite nonzero value at this initial source position. Hence, there is no spatial discontinuity of the retro-propagated field. It has been shown [5] that the TR field has a well-known sinus cardinal form: $\psi_{\mathrm{TR}}(\vec{r},t) \approx \sin_{\mathrm{c}}(kR)$, (where $R = \|\vec{r}-\vec{r}_0\|$ and $k = 2\pi/\lambda$ is the propagation constant). This means that the TR field has a maximum value at the initial source position ($\vec{r}_0$) and a λ main lobe width.

So, this *first step* gives a coarse determination of the source distribution. The spatial resolution depends on the transmitted signal wavelength, via the well-known diffraction-limit. In order to determine the exact location and dimensions of the sources, a better reconstruction-resolution is required. Starting from the result of this *first step*, a *second step* is performed in order to improve the resolution.

B. Resolution improvement step

The key to the resolution issue [26], is mathematically found by time-reversing the Helmholtz equation with sources. The ideal TR field would be:

$$\psi_{\mathrm{TR}}^{\mathrm{ideal}}(\vec{r},\vec{r}_0,t) = \psi(\vec{r},\vec{r}_0,-t) = G(\vec{r},\vec{r}_0,-t) * f(-t). \qquad (4)$$

It appears that the TR field obtained after the coarse reconstruction step is composed of a convergent and a divergent wave. In order to correct this wave superposition effect, the divergent wave should be cancelled.

Let $\psi_{\mathrm{TRM}}(\vec{r},t)$ be the theoretically computed field on the TRMs. This field is injected, during the *first step*, in the TRMs, in order to find the sources. So, the solution to our issue is to excite, during the *second step*, the time-reversed initial excitation: $f(-t) \equiv f(T-t)$ ($T > \mathrm{supp}(f) + \tau$, where $\mathrm{supp}(f)$ is the support of *f*, $\tau = d_{\mathrm{TRM\text{-}source}}/c$ and $d_{\mathrm{TRM\text{-}source}}$ is the distance between the source distribution position and the TRMs position). $f(-t)$ is excited in the source probable positions, in the same time as $\psi_{\mathrm{TRM}}(\vec{r},t)$. To find these positions, we locate, after the *first step*, the maximums of the field amplitude. This time-reversed initial excitation acts like a sink that absorbs the convergent waves during the collapse and cancels the divergent wave.

IV. RESULTS

One of our objectives was to reconstruct EM point-like sources placed in a homogeneous, non-dispersive, and lossless free space. The approach is applied in its simplest form and environment because we want to study the phenomena without introducing any additional dispersion. This application allows a first validation of the proposed reconstruction procedure.

The TLM mesh was excited with a wideband signal, having a central frequency of 30GHz (Fig. 1b). The amplitude-phase distribution of the radiated field on the cubical TRC is used as initial information. A spatial step size of Δl=0.1λ=1mm is chosen for the TLM mesh.

It is important to emphasize that our objective is to realize the TR from a desired radiation pattern and not from real-time signals recorded after the direct approach. In order to get closer to this idea, we use as a starting point of our simulations, for each node of TRMs, only two values: maximum amplitude sample and its correspondent delay.

The described procedure is first applied in order to reconstruct a point-like source of 1mm in diameter, placed in the middle of the cubical TRC of (10x10x10)cm^3 (Fig. 3a). Reconstructed E_z field component amplitude is represented, in the plane containing the source, during the TR (Fig. 4a-d).

We observe that the divergent wave is cancelled after the second step (Fig. 4e-h), with *f*(-*t*) plotted in Fig. 3b. A λ/10 resolution of the reconstruction is obtained (Fig. 5).

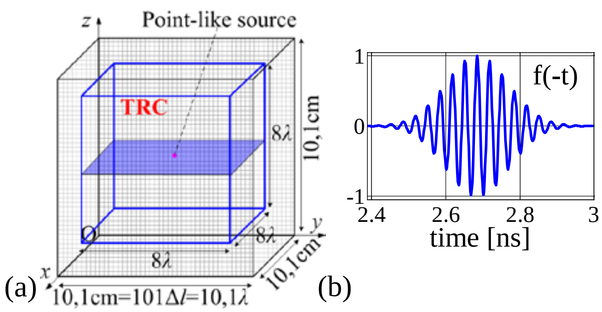

Fig. 3. (a) Point-like source; (b) the sink: *f*(-*t*).

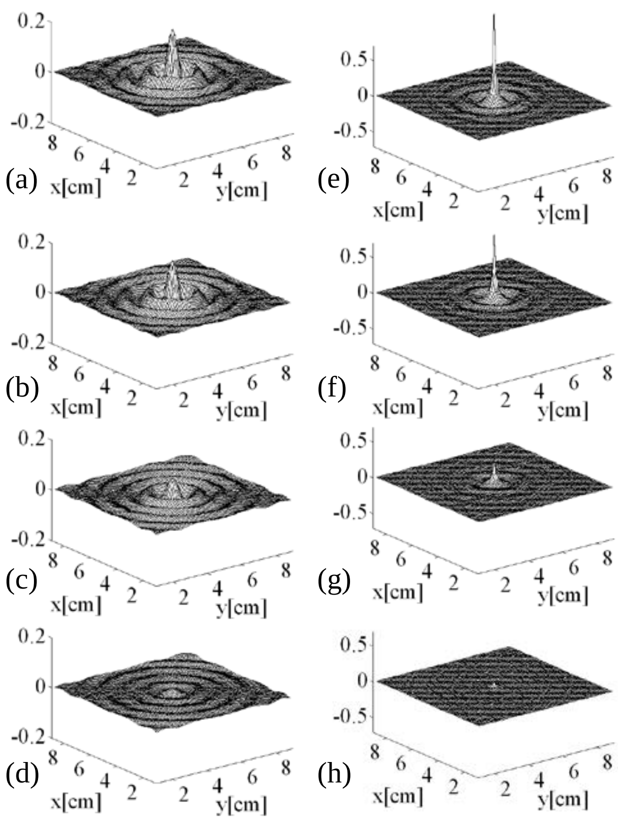

Fig. 4. (a)-(d) - reconstructed source after the *first step*; (e)-(h) - reconstructed source after the *second step*. Process was stopped at 0.366ns, 0.433ns, 0.5ns, 0.566ns.

The same procedure is then applied to reconstruct two identical point-like sources, each of 1mm in diameter and spaced 2λ between centres (Figs. 6a, b). Reconstructed E_z field component amplitude is represented in the plane containing the sources, after the TR (Figs. 7a, b). A good reconstruction resolution of the two sources is achieved in this case as well. The reconstruction of multiple point sources, with any phase delay, can be done from the knowledge of the amplitude- and phase- distribution on the TRCs external surface. The reconstruction of two point sources, excited by two signals in phase opposition, is represented in Fig. 8a.

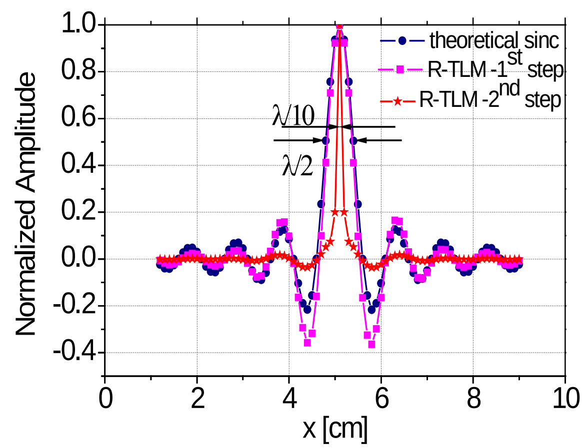

Fig. 5. Focal spot cross-section.

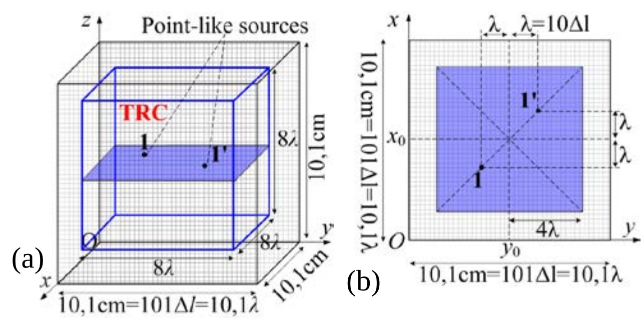

Fig. 6. Two point-like sources: (a) TRC 3D; (b) cross-section.

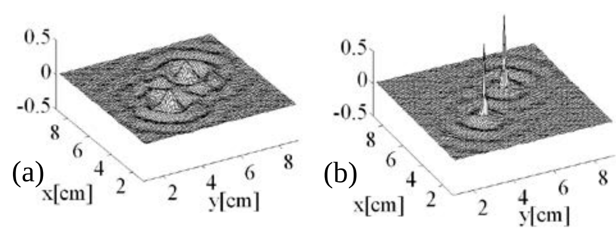

Fig. 7. Reconstructed sources: (a) after the *first step*; (b) after the *second step*.

Hence, we can reconstruct multiple point-like sources, provided that the distance between each of them is at least half the wavelength of the excitation. The reconstruction of a line-like source of 9mm in length is represented in Fig. 8b. The process leads to a widened asymmetrical focal spot. The field obtained after the TR shows an amplitude equalization problem at the initial source positions. The study of more complex sources is under development. Further on, this approach can be used to reconstruct other source distribution shapes or RF devices, e.g. antennas.

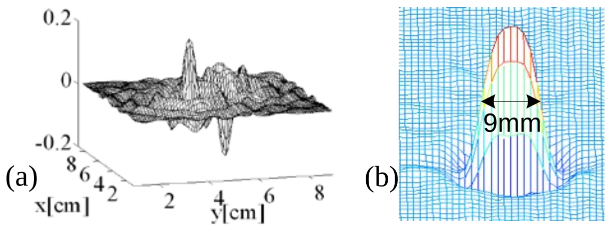

Fig. 8. Reconstruction result of: (a) two point sources in phase opposition; (b) a line-like source.

V. CONCLUSIONS

In this paper, we introduced a new method of EM source synthesis based on wave TR. This approach employs the *Reversed*-TLM method and permits the reconstruction of a source distribution, from its far field radiation.

We showed that it is indeed possible to reconstruct EM point-like sources from their radiated far field. Our proposed *two-step* method begins with a coarse reconstruction step, which gives the approximate positions and dimensions of the sources. The results of this *first step* depend on the excitation wavelength. A *second step* is thus performed in order to increase the resolution of the reconstruction.

Results show that after the resolution improvement step, the diffraction limit is exceeded. These encouraging results let us conclude that the Reversed-TLM method may provide a useful and important new tool for EM source synthesis.

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