Iterative Solution to the Multiple Scattering by A System of Two Infinitely Long Conducting Strips

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Abstract

An analytic solution to the problem of a plane electromagnetic wave scattering by two infinitely long conducting strips is presented using an iterative procedure to account for the multiple scattered field between the strips. To compute the higher order terms of the scattered fields, the translation addition theorem for Mathieu functions is implemented to express the field scattered by one strip in terms of the elliptic coordinate system of the other strip in order to impose the boundary conditions. Scattered field coefficients of high order fields are obtained and written in matrix form. Numerical results are plotted for the scattered in far zone for different strip widths, electrical separations and angles of incidence.

1. Introduction

The multiple scattering of a plane electromagnetic wave by a system of infinitely long conducting strips is important in a variety of practical applications. For example, the solution may be used to study the scattering by complex bodies modeled by a collection of strips, prediction of radiation from elliptical reflector antennas, and to check the accuracy of the results of numerical and approximate methods [1].
Exact analytic solution of the problem of scattering by a system of *N* conducting strips has been formulated using the translation addition theorem for Mathieu functions to enforce the boundary condition [1]. The required computer time and memory to invert the resulting system of matrix increase rapidly with the number of strips. In addition, numerical results for certain strips dimensions, electrical separations and angles of incidence are difficult to obtain by this analytical method may be due to the associated ill-condition system matrices.
In the present paper an iterative procedure is proposed to the scattering by an arbitrary oriented two infinitely long conducting strips. This approach requires the solution of the scattered field by each strip, assumed to be alone in the incident field that acts as an incident field on the other strip. Therefore, the first order scattered field results from the excitation of each strip by the incident field only, while the second order scattered field results from the excitation of each strip by the first order scattered field. Hence, this iterative procedure continues until the solution convergence. One of the advantages of the iterative procedure is that the proposed solution does not require matrix inversion and therefore the desired scattered field coefficients are obtained after each iteration and used in the subsequent iteration.
The solution of the electromagnetic scattering by a system of *N* infinitely long conducting strips has received little attention in the literature due to the complexity of computing Mathieu functions of higher orders and its associated translation addition theorem. Recently, there have been many studies on the multiple scattering by strips [1], circular or elliptic cylinders [2]-[6], spheres [7], and spheroids [8], [9] using different techniques.
Numerical results showing the number of scattered fields are plotted for the normalized echo pattern width with various electrical separations, widths, angles of incidence, and also compared with published results to demonstrate the efficiency of the method [1].

2. Formulation of the problem

Fig. 1 shows the scattering geometry of two infinitely long conducting strips with different widths and arbitrary orientation. The center axes of the two strips are assumed to be parallel to the z-axes. The first strip is located at the origin o_1 while the second strip is located at the polar coordinate point (d, γ) with respect to the global coordinate system (x,y,z). The width of the strips are a_1 and a_2 respectively, and each strip's local coordinate system makes angle α_1 for the first strip and α_2 for the second strip with its global coordinate system. Consider elliptic coordinate systems *u, v,* and *z* such that

$$x = F\cosh u \cos v, \quad y = F\sinh u \sin v, \quad z = z \tag{1}$$

where F is the semifocal length, $0 \le u < \infty$, $0 \le v < 2\pi$, and $-\infty \le z < \infty$. It is usually convenient to introduce

$$\xi = \cosh u \quad , \quad \eta = \cos v \tag{2}$$

with $1 \le \xi \le \infty$ and $-1 \le \eta \le 1$.

Consider the case of a linearly polarized electromagnetic plane wave incident on the two infinitely long conducting strips at an angle ϕ_i with respect to the positive x axis, as shown in Fig. 1, with e^{jwt} time dependence. The electric field component of the TM polarized plane wave of amplitude E_0 is given by

$$E_z^i = E_0 e^{jk\rho\cos(\phi-\phi_i)} \quad (3)$$

where k is the wave number in free space. The incident electric field may be expressed in terms of Mathieu functions about the origins o_1 and o_2 and as follows [10]

$$E_{1z}^i = \sum_{m=0}^{\infty} A_{1em} R_{em}^{(1)}(c_1,\xi_1) S_{em}(c_1,\eta_1) + \sum_{m=1}^{\infty} A_{1om} R_{om}^{(1)}(c_1,\xi_1) S_{om}(c_1,\eta_1) \quad (4)$$

$$E_{2z}^i = \sum_{m=0}^{\infty} A_{2em} R_{em}^{(1)}(c_2,\xi_2) S_{em}(c_2,\eta_2) + \sum_{m=1}^{\infty} A_{2om} R_{om}^{(1)}(c_2,\xi_2) S_{om}(c_2,\eta_2) \quad (5)$$

where

$$A_{1\substack{em\\om}} = E_0 j^m \frac{\sqrt{8\pi}}{N_{\substack{em\\om}}(c_1)} S_{\substack{em\\om}}(c_1, \cos\phi_i^1) \quad (6)$$

$$A_{2\substack{em\\om}} = E_0 j^m \frac{\sqrt{8\pi}}{N_{\substack{em\\om}}(c_2)} S_{\substack{em\\om}}(c_2, \cos\phi_i^2)\, e^{jkd\cos(\gamma-\phi_i)} \quad (7)$$

$$N_{\substack{em\\om}}(c_1) = \int_0^{2\pi} [S_{\substack{em\\om}}(c_1,\eta_1)]^2 dv \quad (8)$$

$$N_{\substack{em\\om}}(c_2) = \int_0^{2\pi} [S_{\substack{em\\om}}(c_2,\eta_2)]^2 dv \quad (9)$$

$$\phi_i^1 = \phi_i - \alpha_1, \phi_i^2 = \phi_i - \alpha_2 \quad (10)$$

and $c_1 = k F_1$, $c_2 = k F_2$, S_{em} and S_{om} are the even and odd angular Mathieu functions of order m, respectively, $R_{em}^{(1)}$ and $R_{om}^{(1)}$ are the even and odd radial Mathieu functions of the first kind, and N_{em} and N_{om} are the even and odd normalized functions.

The scattered electric field from the conducting strips can be expressed in terms of Mathieu functions as

$$E_{1z}^s = \sum_{m=0}^{\infty} B_{em} R_{em}^{(4)}(c_1,\xi_1) S_{em}(c_1,\eta_1) + \sum_{m=1}^{\infty} B_{om} R_{om}^{(4)}(c_1,\xi_1) S_{om}(c_1,\eta_1) \quad (11)$$

$$E_{2z}^s = \sum_{m=0}^{\infty} C_{em} R_{em}^{(4)}(c_2,\xi_2) S_{em}(c_2,\eta_2) + \sum_{m=1}^{\infty} C_{om} R_{om}^{(4)}(c_2,\xi_2) S_{om}(c_2,\eta_2) \quad (12)$$

where B_{em}, C_{em}, B_{om}, and C_{om} are the unknown even and odd scattered field expansion coefficients, and $R_{em}^{(4)}$ and $R_{om}^{(4)}$ are the even and odd Mathieu functions of the fourth kind.

3. First Order Scattered Field by Strips

The first order scattered field results from the separate excitation of each strip by the incident plane wave alone. The boundary condition at the surface of first strip requires the tangential components of the total electric field to be zero, i.e.,

$$\sum_{m=0}^{\infty} A_{1em} R_{em}^{(1)}(c_1,1) S_{em}(c_1,\eta_1) + \sum_{m=1}^{\infty} A_{1om} R_{om}^{(1)}(c_1,1) S_{om}(c_1,\eta_1) + \sum_{m=0}^{\infty} B_{em}^1 R_{em}^{(4)}(c_1,1) S_{em}(c_1,\eta_1) + \sum_{m=1}^{\infty} B_{om}^1 R_{om}^{(4)}(c_1,1) S_{om}(c_1,\eta_1) = 0 \quad (13)$$

where B_{em}^1 and B_{om}^1 are the first order scattered field expansion coefficients. A similar equation may be written corresponds to the second strip. Using the orthogonality properties of the angular Mathieu function yields the first order scattered field coefficients, which may be written for each strip in matrix form as

$$\begin{bmatrix} B_{em}^1 \\ B_{om}^1 \end{bmatrix} = \begin{bmatrix} Q_{enm}^{11} & 0 \\ 0 & Q_{onm}^{11} \end{bmatrix} \begin{bmatrix} A_{1em} \\ A_{1om} \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} C_{em}^1 \\ C_{om}^1 \end{bmatrix} = \begin{bmatrix} Q_{enm}^{22} & 0 \\ 0 & Q_{onm}^{22} \end{bmatrix} \begin{bmatrix} A_{2em} \\ A_{2om} \end{bmatrix} \quad (15)$$

where C^1_{em} and C^1_{om} are the first order scattered field coefficients of the strip, and

$$Q^{11}_{enm} = \frac{R^{(1)}_{en}(c_1,1)}{R^{(4)}_{en}(c_1,1)} \quad , \quad Q^{11}_{onm} = \frac{R^{(1)}_{on}(c_1,1)}{R^{(4)}_{on}(c_1,1)} \tag{16}$$
$$= 0\ ,\ n \neq m \quad , \quad = 0\ ,\ n \neq m$$

Similar equations may be written correspond to Q^{22}_{enm} and Q^{22}_{onm}.

4. Higher Order Scattered Field by Strips

The second order field results from the excitation of each strip by the scattered field from the other strip due to the initial incident field. The boundary condition at the surface of first strip requires the tangential components of the total electric field to be zero, i.e,

$$\begin{aligned} &\sum_{m=0}^{\infty} C^1_{enm}\ R^{(4)}_{em}(c_2,1)S_{em}(c_2,\eta_2) \\ &+ \sum_{m=1}^{\infty} C^1_{onm}\ R^{(4)}_{om}(c_2,1)S_{om}(c_2,\eta_2) \\ &+ \sum_{m=0}^{\infty} B^2_{em} R^{(4)}_{em}(c_1,1)S_{em}(c_1,\eta_1) \\ &+ \sum_{m=1}^{\infty} B^2_{om}\ R^{(4)}_{om}(c_1,1)S_{om}(c_1,\eta_1) = 0 \end{aligned} \tag{17}$$

where B^2_{em} and B^2_{om} are the second order scattered field expansion coefficients of the first strip. To enforce the boundary condition, the first order scattered field from the second strip must be expressed in terms of the coordinate systems of the first strip by using the addition theorem for the Mathieu functions [11], i.e.,

$$\begin{aligned} R^{(4)}_{\substack{em\\om}}(c_2,\xi_2)S_{\substack{em\\om}}(c_2,\eta_2) &= \sum_{l=0}^{\infty} WE^{2\to1}_{\substack{elm\\olm}} R^{(1)}_{el}(c_1,\xi_1)S_{el}(c_1,\eta_1) \\ &+ \sum_{l=1}^{\infty} WO^{2\to1}_{\substack{elm\\olm}} R^{(1)}_{ol}(c_1,\xi_1)S_{ol}(c_1,\eta_1) \end{aligned} \tag{18}$$

where

$$WE^{2\to1}_{\substack{elm\\olm}} = \frac{\pi\, j^{l-m}}{N_{e1}(c_1)} = \sum_{i=0}^{\infty}\sum_{p=0}^{\infty} (-j)^{i+p} D^m_{\substack{ei\\oi}}(c_2) D^l_{ep}(c_1) X^{2\to1}_{\substack{eip\\oip}} \tag{19}$$

$$WO^{2\to1}_{\substack{elm\\olm}} = \mp \frac{\pi\, j^{l-m}}{N_{o1}(c_1)} = \sum_{i=0}^{\infty}\sum_{p=0}^{\infty} (-j)^{i+p} D^m_{\substack{ei\\oi}}(c_2) D^l_{op}(c_1) Y^{2\to1}_{\substack{eip\\oip}} \tag{20}$$

and

$$X^{2\to1}_{\substack{eip\\oip}} = H^{(2)}_{p-i}(kd)\begin{bmatrix}\cos\Psi^-\\ \sin\Psi^-\end{bmatrix} + (-1)^i H^{(2)}_{p+i}(kd)\begin{bmatrix}\cos\Psi^+\\ \sin\Psi^+\end{bmatrix} \tag{21}$$

$$Y^{2\to1}_{\substack{eip\\oip}} = H^{(2)}_{p-i}(kd)\begin{bmatrix}\sin\Psi^-\\ \cos\Psi^-\end{bmatrix} - (-1)^i H^{(2)}_{p+i}(kd)\begin{bmatrix}\sin\Psi^+\\ \cos\Psi^+\end{bmatrix} \tag{22}$$

with

$$\Psi^+ = i\Psi_{21} + p\Psi_{12}\ ,\ \Psi^- = i\Psi_{21} - p\Psi_{12}\ . \tag{23}$$

In the above equations, Ψ_{12} and Ψ_{21} are measured from the local positive x axis of each strip to the separation distance between the strips, $H^{(2)}_{p+i}(kd)$ is the Hankel function of the second kind with argument kd, and D^n_{ej} and D^n_{oj} are the Fourier coefficients of the Mathieu functions [10]. The sum is over only even or odd values of *i(p)* depending whether m(*l*) is even or odd in equations (19) and (20). Substituting equation (18) into (17) and using the orthogonality properties of the angular Mathieu functions yields the second order scattered field coefficients, which may be written for each strip in matrix form as

$$\begin{bmatrix}B^2_{em}\\ B^2_{om}\end{bmatrix} = \begin{bmatrix}Q^{11}_{enm} & 0\\ 0 & Q^{11}_{onm}\end{bmatrix}\begin{bmatrix}Q^{12}_{eenm} & Q^{12}_{eonm}\\ Q^{12}_{oenm} & Q^{12}_{oonm}\end{bmatrix}\begin{bmatrix}C^1_{em}\\ C^1_{om}\end{bmatrix} \tag{24}$$

$$\begin{bmatrix}C^2_{em}\\ C^2_{om}\end{bmatrix} = \begin{bmatrix}Q^{22}_{enm} & 0\\ 0 & Q^{22}_{onm}\end{bmatrix}\begin{bmatrix}Q^{21}_{eenm} & Q^{21}_{eonm}\\ Q^{21}_{oenm} & Q^{21}_{oonm}\end{bmatrix}\begin{bmatrix}B^1_{em}\\ B^1_{om}\end{bmatrix} \tag{25}$$

where C^2_{em} and C^2_{om} are the second order scattered field expansion coefficients of the second strip, and

$$Q^{12}_{eenm} = WE^{2\to1}_{enm}\ ,\ Q^{12}_{eonm} = WE^{2\to1}_{onm}\ ,\ Q^{12}_{oenm} = WO^{2\to1}_{enm}\ ,$$
$$\text{and } Q^{12}_{oonm} = WO^{2\to1}_{onm} \tag{26}$$

Similar equations may be written correspond to Q^{21}_{eenm}, Q^{21}_{eonm}, Q^{21}_{oenm}, and Q^{21}_{oonm}.

To obtain a general solution, we solve similarly for the higher order scattered fields which are sensitive to the electrical widths, separation between the strips and angles of incidence. This means if the strips are located very close to one another, then the higher order scattered fields are significant and therefore should be included in the solution. The significance of the higher order scattered fields will be verified numerically by comparison with published data.

The general expression for the *k*th order scattered field coefficients may be written as

$$\begin{bmatrix}B^k_{em}\\ B^k_{om}\end{bmatrix} = \begin{bmatrix}Q^{11}_{enm} & 0\\ 0 & Q^{11}_{onm}\end{bmatrix}\begin{bmatrix}Q^{12}_{eenm} & Q^{12}_{eonm}\\ Q^{12}_{oenm} & Q^{12}_{oonm}\end{bmatrix}\begin{bmatrix}C^{k-1}_{em}\\ C^{k-1}_{om}\end{bmatrix} \tag{27}$$

$$\begin{bmatrix} C_{em}^{k} \\ C_{om}^{k} \end{bmatrix} = \begin{bmatrix} Q_{enm}^{22} & 0 \\ 0 & Q_{onm}^{22} \end{bmatrix} \begin{bmatrix} Q_{eenm}^{21} & Q_{eonm}^{21} \\ Q_{oenm}^{21} & Q_{oonm}^{21} \end{bmatrix} \begin{bmatrix} B_{em}^{k-1} \\ B_{om}^{k-1} \end{bmatrix}. \qquad (28)$$

It should be noted that the matrices in equations (27) and (28) are computed once (i.e., k=2) for the electrical sizes and separation considered and used for the subsequent iterations (i.e., k=3,4,…).

Once the scattered field coefficients are determined, the total far field from the strips due to the kth order scattered field can be determined [1]-[5].

5. Numerical Results

In order to solve for the unknown scattered field coefficients, the infinite series are first truncated to include only the first N terms, where N in general, is a suitable truncation number proportional to the strips electrical width. In the computation, the value of N has been chosen to impose a convergence condition that provides solution accuracy with at least four significant figures [14], [15]. It is found that increasing the electrical width of the scatterers will increase the total truncation number of N terms [16].

To check the accuracy of our computer program, we recomputed first the results given in references [2], [12] for large electrical separation when it is compared with the electrical sizes of the scatteres and we obtained complete agreement between methods by only implementing the first order scattered field using the iterative solution. Fig. 2 shows the numerical result of the normalized echo width pattern $\sqrt{\sigma/\lambda}$ versus the scattering angle ϕ for two identical strips with electrical width ka=3.14. The electrical separation between the center of the strips is assumed to be kd=12.5 and at an angle of incidence $\phi_i = 90^o$ (broadside incidence). It can be seen that the results of the first scattered order (k=1) presented by solid line is satisfactory at all backscattering angles because the electrical separation between the strips is large compared to their width. To set a criterion for terminating the iteration process, the scattered field after each iteration is calculated and divided by the total field scattered from the pervious iterations, and the process is terminated when the ratio is smaller than 10^{-4} [7]. Fig. 3 has the same electrical parameters except the electrical separation is reduced to 7. It can be seen that the numerical results of the first order scattered field is satisfactory except at resonance scattering angles. This is because the first order scattered field does not take into account the interaction between the strips and hence k=1 represents the sum of the scattered field due to the incident field only. The significance of the multiple scattered fields can be seen in the second scattered order term ($k=2$) which includes the scattered fields due to the plane wave incidence plus the scattered fields due to the first order scattered field due to the incident field on each strip. However, the results show that four scattered field orders are needed to obtain convergent solution at the resonance scattering angles. Fig. 4 is similar to Fig. 2 except the width of the second strip is reduced from 3.14 to 2.0 and kd=5.5. We can see that the number of scattered fields needed is four to obtain convergent solution. Fig. 5 shows the normalized echo width pattern for two identical strips of width ka=5.0, kd=13, and at angle of incidence of zero degree (endfire). Three iterations are needed to obtain convergent solution. Fig. 6 is similar to Fig. 5 except that the incident angle is 90 degrees and kd=11.

Fig. 7 shows the numerical results of the normalized backscattering echo width pattern versus the electrical separation (kd) for two identical strips of width ka=5.0 and at angle of incidence of zero degree. The electrical separation is taken between 11 and 23. The results show that the behavior of the backscattering cross section is sinusoidally and with k=4 a convergent solution is obtained at all electrical separations. Fig. 8 is similar to Fig. 7 except the incident angle is 90 degrees. Again, the backscattering cross section is behaving sinusoidally and four scattered field orders is needed to obtain convergent solution.

6. Conclusions

We have investigated the problem of multiply field scattered due to a plane electromagnetic wave incident on arbitrary oriented two perfectly conducting strips. The boundary conditions were implemented using the translation addition theorem. The numerical results indicated that the number of multiple scattered fields depends on the electrical width of the strips, electrical separations and incident angles. We have seen that the iterative solution gives insight to the nature of the multiple scattered fields where it is sometime strong (more terms needed, Fig. 3 at ϕ 88°) or weak (less terms needed, Fig. 3 at ϕ 200°) at some specific scattering angles. A potential advantage of using the iterative solution is that of saving computer time and memory by avoiding the inversion of system matrix.

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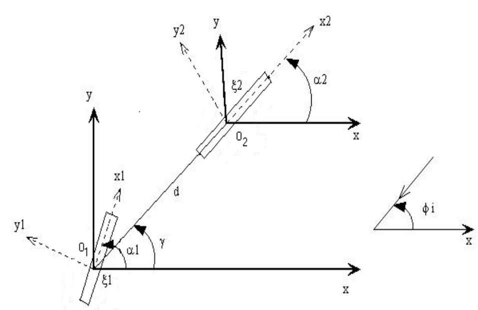

Figure 1: Scattering geometry of two conducting strips.

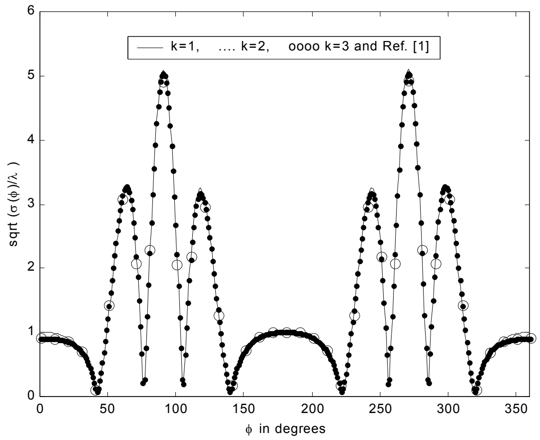

Figure 2: Normalized echo pattern width versus the scattering angle ϕ for two identical conducting strips with ka1=ka2=3.14, kd=12.5, $\alpha_1 = \alpha_2 = 0^o$, $\phi_i = 90^o$, $\gamma = 0^o$.

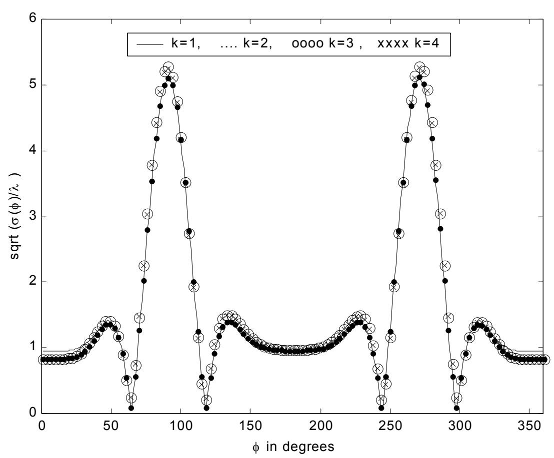

Figure 3: Normalized echo pattern width versus the scattering angle ϕ for two identical conducting strips with ka1=ka2=3.14, kd=7, $\alpha_1 = \alpha_2 = 0^o$, $\phi_i = 90^o$, $\gamma = 0^o$.

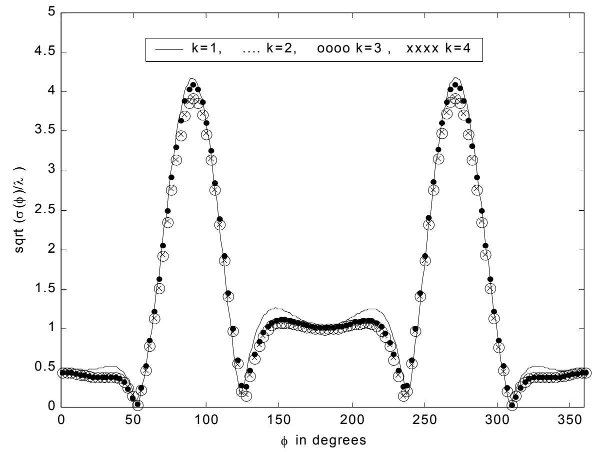

Figure 4: Normalized echo pattern width versus the scattering angle ϕ for two conducting strips with ka1=3.14 , ka2=2.0, kd=5.5, $\alpha_1 = \alpha_2 = 0^o$, $\phi_i = 90^o$, $\gamma = 0^o$.

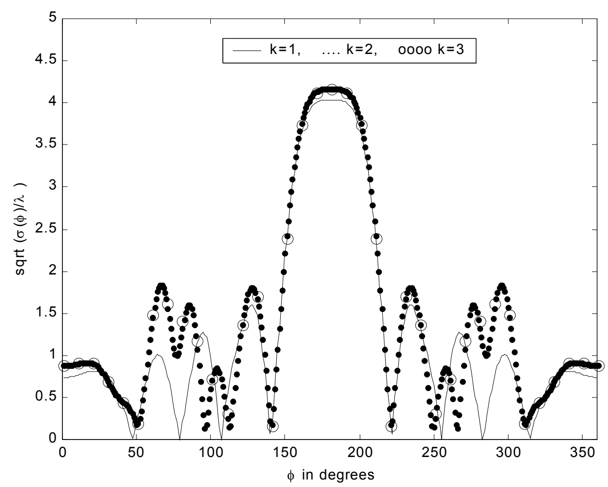

Figure 5: Normalized echo pattern width versus the scattering angle ϕ for two conducting strips with ka1=ka2=5.0, kd=13, $\alpha_1 = \alpha_2 = 0^o$, $\phi_i = 0^o$, $\gamma = 0^o$.

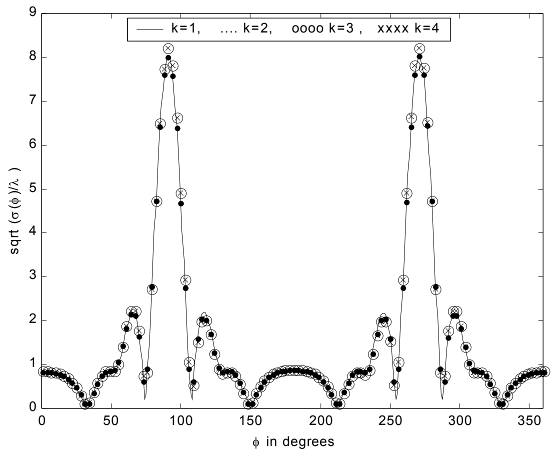

Figure 6: Normalized echo pattern width versus the scattering angle ϕ for two conducting strips with $ka_1=ka_2=5.0$, kd=11, $\alpha_1=\alpha_2=90^o$, $\phi_i=0^o$, $\gamma=0^o$.

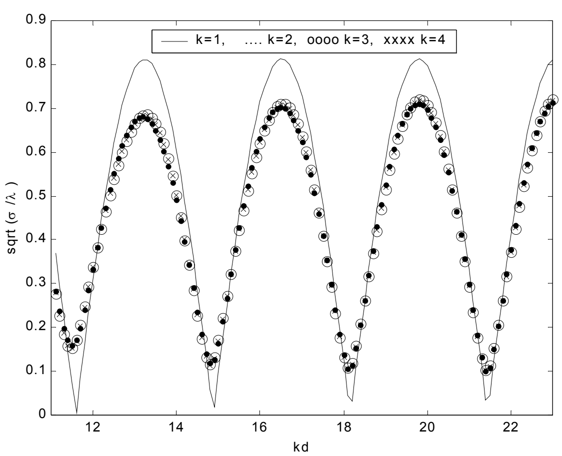

Figure 7: Normalized backscattering cross section versus the electrical separation kd for conducting strips with $ka_1=ka_2=5$, $\alpha_1=\alpha_2=0^o$, $\phi_i=0^o$, $\gamma=0^o$.

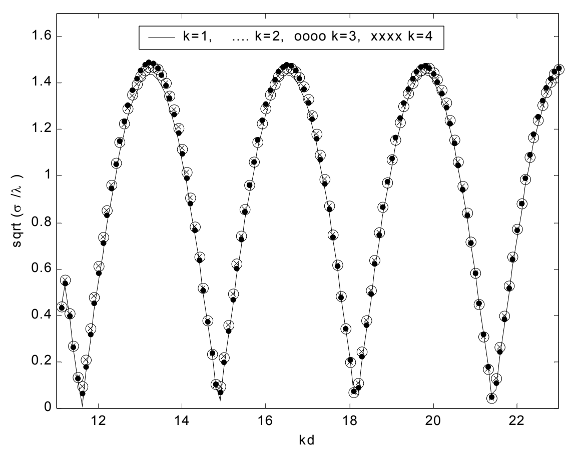

Figure 8: Normalized backscattering cross section versus the electrical separation kd for conducting strips with $ka_1=ka_2=5$, $\alpha_1=\alpha_2=0^o$, $\phi_i=90^o$, $\gamma=0^o$.

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