ANALYSIS OF A SUSPENDED STRIP IN A CIRCULAR CYLINDRICAL WAVEGUIDE

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Abstract: The separation of variables method along with transformation theorem form Mathieu functions to Bessel functions are employed here to analyze the problem of a suspended strip in a circular waveguide. An infinite dimensional determinant is obtained which represents the characteristic equation of the proposed structure. To obtain the cutoff wavenumbers for both TE and TM cases of such a structure, the infinite determinant is truncated and convergence was observed. Numerical results for cases of interest are then presented.

Introduction

The cut-off frequencies of circular cylindrical waveguides loaded with eccentric inner conductors were extensively investigated. For instance, a general problem of a conducting cylinder placed inside a conducting hollow tube of arbitrary cross section was formulated and solved by the point-matching technique in [1]. Kuttler [2] obtained the lower and upper bounds of the cutoff wavenumbers using different methods. Analytical formulations were also developed and used to calculate exact values of the cutoff wavenumbers in [3-5] employing different methodologies. Recently, Das and Vargheese employed a bilinear transformation to transform the two-wire and the eccentric transmission line into the concentric coaxial configuration [6].

Elliptical waveguides have been the subject of many investigations due to their wide applications in radar feed lines, multichannel communication and accelerator beam tubes. Another line of research [7] investigated elliptical waveguide loaded with ridges or a suspended strip. It was assumed that the ridges extended from the walls to the focal points. Recently Rozzi et al. [8] reported a complete analysis for a suspended strip in an elliptical cylindrical waveguide. They considered the case where the suspended strip extended between the focal points of the elliptical waveguide. He obtained the cutoff wavelengths for different TEM, TE and TM modes using the separation of variables. An extension to Rozzi's analysis for the more general case of a strip of arbitrary width was reported in [9]. The strip width could be larger or smaller than the focal length of the elliptic cylinder.

The special case of suspended strip in a circular cylindrical waveguide has not been yet addressed. Results for such a special case can not be directly obtained from the general case reported in [9] due to singularity of the Mathieu functions when the outer ellipse of the waveguide has zero focal length. Meanwhile if the focal length is considered very small the resulting cutoff wavenumbers are not close to those of the special case because they are very sensitive to the geometrical dimensions. Accordingly, such a special case has to be treated separately. The problem involves two different coordinate systems, i.e. circular cylindrical (to fit the circular boundary of the outer waveguide) and elliptical coordinates (to fit the elliptical boundary of the strip) which reflects the use of both Bessel and Mathieu functions. This paper addresses such a problem and employs the transformation from Mathieu to Bessel functions [10] to facilitate the application of the boundary conditions. Some special cases are introduced first for comparison with published data to ensure that our program is correct then other new results are introduced.

Theory

Consider the two-dimensional cross-sectional geometry shown in Fig. 1. It consists of an infinitely long perfectly conducting circular waveguide with radius b. A perfectly conducting strip of width $2a$ and infinite length is placed such that its axes coincide with that of the circular waveguide. In order to facilitate our analysis, two coordinate systems are considered. The local elliptical coordinates (u_o, v_o, z) are at the center of the strip while the global circular coordinates

(ρ,φ,z) are considered at the center of the circular cylinder. The solution of the scalar Helmholtz wave equation in terms of elliptical coordinates can be written as

$$\psi(\zeta_o,\eta_o)=\sum_{n=0}^{\infty}\{Ae_n\,Je_n(c_o,\zeta_o)+Be_n\,Ne_n(c_o,\zeta_o)\}\,Se_n(c_o,\eta_o)e^{-j\beta z} \quad (1)$$

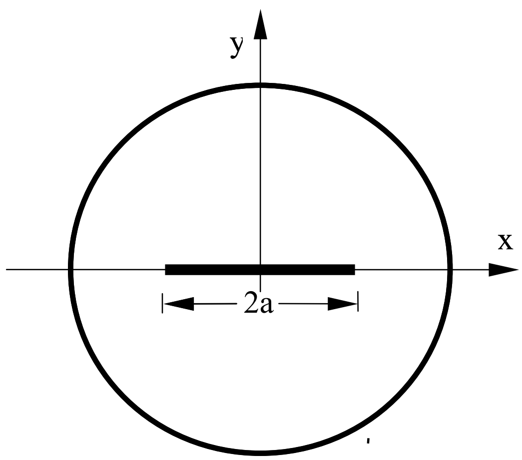

Fig. 1. Geometry of the problem.

for the even modes, and while for the odd modes,

$$\psi(\zeta_o,\eta_o)=\sum_{n=0}^{\infty}\{Ao_nJo_n(c_o,\zeta_o)+Bo_n\,No_n(c_o,\zeta_o)\}So_n(c_o,\eta_o)e^{-j\beta z} \quad (2)$$

where $\zeta=\cosh u$, $\eta=\cos v$, $c_o=k_c a$ ($k_c^2=k^2-\beta^2$, k_c is the transverse component, while $k=2\pi/\lambda$ is the free space wavenumber, λ is the wavelength and β is the propagation constant). Je_n and Ne_n are even modified radial Mathieu functions of the first and second kind, respectively, while Jo_n and No_n are their corresponding odd functions. Se_n and So_n are the even and odd angular Mathieu functions. Ae_n, Ao_n, Be_n and Bo_n are coefficients to be calculated by imposing the boundary conditions.

1. *TE Case*

These modes must satisfy the boundary condition of vanishing tangential components of the electric field ($E_v=0$) on the perfectly conducting surfaces, i. e:

$$\frac{\partial\psi}{\partial u_o}\Big|_{u_o=0}=0,\quad 0\le v_o\le 2\pi \qquad \text{and}$$

$$\frac{\partial\psi}{\partial\rho}\Big|_{\rho=b}=0\quad,\qquad 0\le\phi\le 2\pi. \quad (3)$$

Since we have even and odd modes in each case, one can consider them individually.

(a) *TE Even Modes*

Applying the first boundary condition $\frac{\partial\psi}{\partial u_o}\big|_{u_o=0}=0,\ 0\le v_o\le 2\pi$, one can obtain $Be_n=0$, which when substituting in (1) yields

$$\psi(\zeta_o,\eta_o)=\sum_{n=0}^{\infty}Ae_n\,Je_n(c_o,\zeta_o)\,Se_n(c_o,\eta_o)\,e^{-j\beta z}. \quad (4)$$

Now in order to apply the second boundary condition $\psi(\zeta_o,\eta_o)$ must be transferred to the circular cylindrical coordinate system (ρ,φ,z). This can be done using the addition theorem of Mathieu functions [10], which is simplified for this case as

$$\begin{matrix}Re_p^{(j)}(c_o,\zeta_o)Se_n(c_o,\eta_o)\\ Ro_p^{(j)}(c_o,\zeta_o)So_n(c_o,\eta_o)\end{matrix}=\sqrt{\frac{\pi}{2}}\sum_{l=0}^{\infty}(j)^{l-p}\begin{matrix}De_l^p(c_o)\\ Do_l^p(c_o)\end{matrix}Z_l^{(i)}(k\rho)\begin{matrix}\cos(l\alpha)\\ \sin(l\alpha)\end{matrix} \quad (5)$$

where $Re_p^{(j)}$ could be Je_n or Ne_n while $Ro_p^{(j)}$ is Jo_p or No_p and $Z_l^{(j)}$ is the Bessel function of the first or second kind, respectively. The constants De_m^n and Do_m^n are coefficients of the infinite series of angular Mathieu functions in terms of trigonometric functions defined in [9]. To apply the second boundary condition, (5) is employed in (4) as

$$\psi(\rho,\phi)=\sqrt{\frac{\pi}{2}}\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}(j)^{l-n}\;De_l^n(c_o)\;J_l(k_c\rho)\;\cos(l\phi)\;\;e^{-j\beta z}. \quad (6)$$

Applying the second boundary condition and using the orthogonal property of the trigonometric functions, one obtains

$$\sum_{n=0}^{\infty}Ae_n(j)^{l-n}De_l^n(c_o)\,J_l'(k_cb)\quad l=0,1,2,... \quad (7)$$

Equation (7) can be written in the following matrix form,

$$\left[Z_{l,n}\right]\,\left[Ae_n\right]=0. \quad (8)$$

A non-trivial solution can be obtained if the determinant of the Z matrix vanishes. The solution of the resulting determinant will give the values of k_c corresponding to the 1st, 2nd, … and nth cutoff wavenumbers. Once the value of k_c is obtained for the ith cutoff wavenumber, the coefficients can be obtained and the field distribution inside the waveguide is then completely defined.

(b) *TE odd Modes*

Following the same procedure described for the TE even modes but with $\psi(\zeta_o, \eta_o)$ represented in terms of odd functions [equation (2)], one ends up with a matrix equation similar to that in (8), where the elements of the matrix are given by:

$$Z_{l,n} = (j)^{l-n}\ Do_l^n(c_o)\left\{J_l'(k_c\rho) - \frac{Jo_n'(c_o,1)}{No_n'(c_o,1)}\right. \quad (9)$$
$$\left. -N_l'(k_c\rho)\right\} \qquad l = 1, 2, 3,..$$

and the coefficient vector is denoted as Ao_n. Again the determinant of the matrix is equal to zero to obtain the cutoff wavenumbers.

2. *TM Case*

These modes must satisfy the boundary condition of vanishing tangential components of the electric field ($E_u = 0$) on the perfectly conducting surfaces, i.e.

$$\psi|_{u_o=0} = 0 \quad , \qquad 0 \le v_o \le 2\pi \qquad \text{and}$$
$$\psi\ |_{\rho=b} = 0 \quad , \qquad 0 \le \phi \le 2\pi. \qquad (10)$$

Even and odd modes can be considered individually as follows:

(a) *TM Even Modes*

Applying the first boundary condition along with the orthogonal property of Mathieu functions yields

$$\psi(\zeta_o,\eta_o) = \sum_{n=0}^{\infty} Ae_n \left\{Je_n(c_o,\zeta_o) - \frac{Je_n(c_o,1)}{Ne_n(c_o,1)}\right. \quad (11)$$
$$\left. - Ne_n(c_o,\zeta_o)\right\} Se_n(c_o,\eta_o)\, e^{-j\beta z}.$$

Employing the addition theorem of the Mathieu functions and applying the second boundary condition along with the orthogonal property of the triangular Mathieu functions, one can get matrix equation similar to (8) with the elements of Z matrix given by

$$Z_{l,n} = (j)^{l-n}\ De_l^n(c_o)\left\{J_l(k_c b) - \frac{Je_n(c_o,1)}{Ne_n(c_o,1)}\right. \quad (12)$$
$$\left. - N_l(k_c b)\right\} \qquad l = 0, 1, 2,...$$

(b) *TM odd Modes*

Following the same procedure described for the TM even modes but with $\psi(\zeta_o, \eta_o)$ represented in terms of odd functions [equation (2)], one ends up with a matrix equation similar to that in (8), where the elements of the matrix are given by:

$$Z_{l,n} = (j)^{l-n}\ Do_l^n(c_o) J_l(k_c b) \qquad l = 1, 2,... \quad (13)$$

and the coefficient vector is denoted as Ao_n.

Results and Discussion

To check the accuracy of our computations the cutoff wavelengths of the special case of circular waveguide is considered by assuming the strip width very small. The results obtained agreed very well with those published in [12].

For the general case, the effect of two parameters *a* and *b* (strip width and circular cylinder radius) on the cutoff wavelength was studied. The first cutoff wavenumber versus the strip width for different values of circular guide radius is illustrated in Fig. 2 for both even and odd TM modes. As one can see from Fig. 2(a), the cutoff wavenumber of the even mode increases when the strip width increases. On the other hand, one can see that for the odd TM case, the cutoff wavenumber is constant for all values of the strip width, which shows that the strip has no effect on the cutoff wavenumber for such a mode. This can be explained if one looks at the field distribution of the circular waveguide for the TM_{11} mode [12], where the electric field has no tangential component along one of the circular cylinder diameters at which the strip is going to be placed.

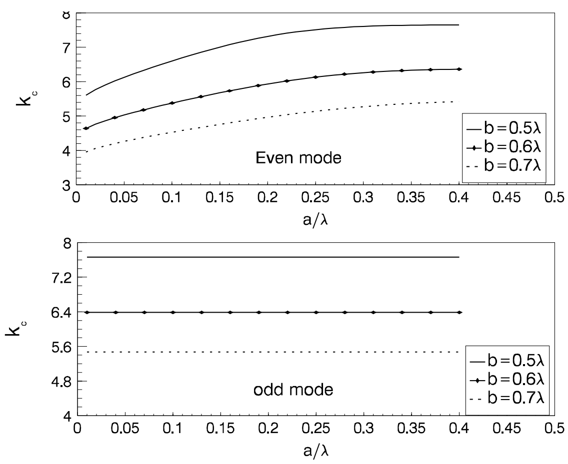

Fig. 2. The first cutoff wavelength versus strip width for even and odd TM modes.

Figure 3 illustrates the effect of the strip width, on the cutoff wavenumber for both even and odd TE modes for different values of circular guide radius. The cutoff wavenumber for the TE case is constant for even modes (Fig. 3(a)) and decreases with the increase of the strip width for odd modes, as shown in Fig. 3(b).

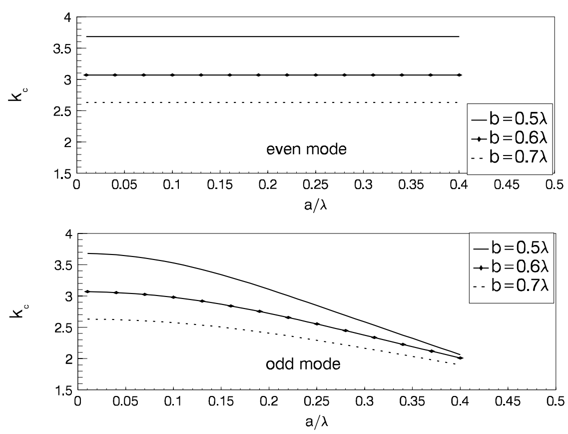

Fig. 3. The first cutoff wavelength versus strip width for even and odd TE modes.

The constant values of the cutoff wavenumbers for the even TE mode can also be realized when one considers the field distribution of such a mode for unloaded circular waveguide. The second cutoff wavenumber was also calculated for both odd and even TE and TM modes as illustrated in Fig. 4 and Fig. 5, respectively. Similar behavior for both TE and TM modes were found except that the rate of increase or decrease of the cutoff wavenumber with the strip width is lower than that corresponding to the first cutoff wavenumber.

Conclusion

The cutoff wavenumbers for both TE and TM cases of an arbitrary strip width suspended in a circular cylindrical waveguide have been calculated. It is found that the cutoff wavenumber increases with the strip width for even TM modes and decreases for odd TE modes.

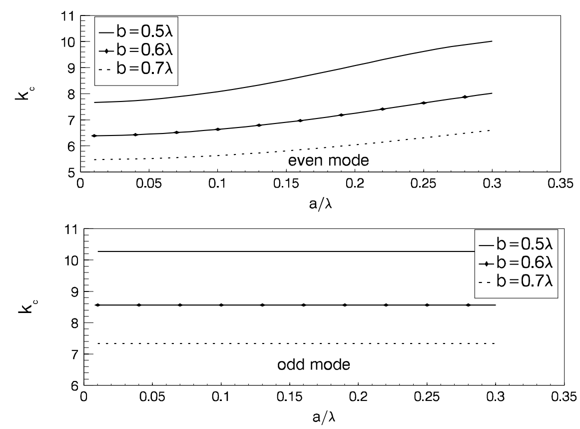

Fig. 4. The second cutoff wavelength versus strip width for even and odd TM modes.

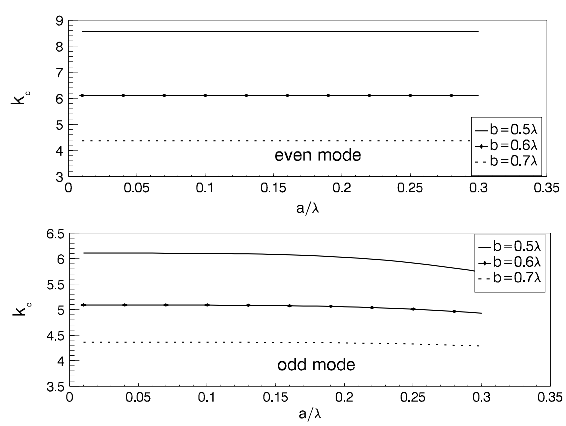

Fig. 5. The second cutoff wavelength versus strip width for even and odd TE modes.

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