Response Bounds Analysis for Transmission Lines Characterized by Uncertain Parameters

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Abstract

This work is focused on the study of Multiconductor Transmission Lines (MTL) with uncertain parameters; i.e. the values of r, l, c and g can vary in an interval. The wavelet expansion in time domain is used in order to obtain an accurate and low cost representation of the line in terms of an algebraic system. The wavelet representation applied to the study of MTL with variation of the electrical parameters allow us to easily calculate a set of equivalent distributed generators, which represent the effects of the disturbance produced by the parameter variation. This analysis allows us to directly evaluate the response bounds related to the parameters uncertainties without performing repeated simulations (Montecarlo Method).

Keywords— **Uncertain Parameters, Transmission Lines, Time Domain Expansion.**

1 Introduction

The study of the effect of uncertainties in the electrical parameters of MTLs is an important yet complex topic; its importance comes from the fact that even the most developed industrial technologies cannot guarantee 100% accuracy in the construction of electronic devices, where transmission lines play a key role. Furthermore, the aging process is another cause of the parameters variation with respect to the nominal value. The effect of uncertainties can be studied by statistic Montecarlo techniques, that suffer of long computational times [1], by probabilistic approaches under some simplifying hypotheses [2], or by calculating a time domain sensitivity function (see for example [3]).

In this paper, the telegrapher equation is expanded in the wavelet domain; more precisely a time domain wavelet expansion is performed, as in [4], [5]. This technique is chosen because it allows to represent the MTL through a sparse algebraic system, where the unknowns are the wavelet coefficients of voltages and currents, and the system matrix is a function of the electrical parameters of the MTL. The time domain solution is then obtained by simply solving the algebraic system and inverse transforming the results. It is noteworthy that the technique for the bounds definition proposed here, can be applied to any solution technique characterized by the expansion of the time variables on a basis functions, for example as performed in [6], resulting in a linear algebraic system.

This representation and some simple algebraic calculations performed on the system matrix let us calculate a set of equivalent time domain distributed generators, representing the effects of the uncertainties on the nominal output. The analysis of the magnitude of these equivalent distributed generators allows us to understand the effect of each single variation, and most important a simple procedure is defined to determine the response bounds due to the variation of the parameters in the given range. Insights of the physical meaning of the procedure and some results are shown.

2 Wavelet Based Modeling of Multiconductor Transmission Lines

As widely addressed in [4] the use of the Wavelet Expansion (WE) for the simulation of multiconductor transmission lines is a powerful tool, allowing fast and accurate simulations. The way the wavelet based model is obtained is the following: starting from the quasi-TEM MTL equations, the time variation of voltages and currents (which are variable with space and time coordinates) are expanded on a wavelet basis, yielding space variable vectors of coefficients. Time derivatives are dealt with by using the differential (or integral, in case the MTL equations are first integrated) operator in the wavelet domain, which are constant and sparse matrices, calculated only once prior to the simulation.

In the so obtained equations only the space variable appears now; for this reason the line needs to be segmented in a number of cells, each represented by a sparse matrix, which can be cascade connected in order to obtain an algebraic system, which solved gives the value of the wavelet coefficients. The general form of the system is reported in equation (1)

$$\begin{bmatrix} \mathbf{I}_d & \mathbf{Ch} & 0 & \cdots & \cdots \\ 0 & \mathbf{I}_d & \mathbf{Ch} & \cdots & \cdots \\ 0 & 0 & \mathbf{I}_d & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \mathbf{I}_d & \mathbf{Ch} \\ \mathbf{BcL} & \cdots & \cdots & \cdots & \mathbf{Bc0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_L \\ \mathbf{i}_L \\ \mathbf{v}_{L-1} \\ \mathbf{i}_{L-1} \\ \vdots \\ \mathbf{v}_j \\ \mathbf{i}_j \\ \vdots \\ \mathbf{v}_0 \\ \mathbf{i}_0 \end{bmatrix} = \mathbf{E} \quad (1)$$

where

$$\mathbf{E} = [0,0,0,0,\ldots,0,0,\ldots,\mathbf{E}_L,\mathbf{E}_0]^T \quad (2)$$

and $\mathbf{I}_d$ is the identity matrix of the proper dimension; $\mathbf{v}_j$ and $\mathbf{i}_j$ are the vectors of wavelet coefficients of voltages and currents at each cell (in particular $\mathbf{v}_L$, $\mathbf{i}_L$, $\mathbf{v}_0$, $\mathbf{i}_0$ are the voltages and currents, respectively, at the two terminations of the line); **BcL** and **Bc0** are the matrices of the boundary conditions (generators and terminations loads). The matrix **Ch**, which contains the equation of a single cell, is analytically obtained and has the following expression

$$\mathbf{Ch} = \begin{bmatrix} 0 & (-l\mathbf{D} - r\mathbf{I}_d) \\ (-c\mathbf{D} - g\mathbf{I}_d) & 0 \end{bmatrix} \quad (3)$$

where $\mathbf{D}$ is the wavelet representation of the differential operator, as previously discussed, and r, l, c, g are obviously the line parameters. As it can be easily observed, system (1) is an algebraic system characterized by a sparse matrix; hence it can be easily solved by iterative techniques. The known term of the system is characterized by having all zero entries except than the last part, where the WE of the input signals (generator) is included (vectors $\mathbf{E}_0$ and $\mathbf{E}_L$).

Uniform and nonuniform transmission lines (with linear and nonlinear load) can be in this way conveniently solved, obtaining accurate solutions in lower CPU times if compared with standard step by step techniques.

At this stage frequency dependent transmission lines could not be included in the model. As a result of the further work performed by the authors, also frequency dependence of the parameters and proximity effect between the conductors have been included. The inclusion of these two important phenomena are presented in [5], in which starting from the original formulation presented in [7] and expressing the convolution between two functions in the wavelet domain, a convenient formulation is obtained. In particular, the algebraic system representing the MTL is obtained in the form of (1), in which the only difference is the presence of a constant matrix $\mathbf{K}$, function of the skin and proximity effect sensitive quantities (i.e. geometrical and physical characteristics of the line) simply included in the matrix **Ch** as

$$\mathbf{Ch} = \begin{bmatrix} 0 & (-l\mathbf{D} - r\mathbf{I}_d - \mathbf{K}) \\ (-c\mathbf{D} - g\mathbf{I}_d) & 0 \end{bmatrix}. \quad (4)$$

As a conclusion to this section we underline that the use of WE for the solution of such problems, in our formulation allows the fast simulation of uniform, nonuniform, frequency dependent transmission lines; not being a frequency domain based method also nonlinearities can be easily included, as widely addressed in [4] and [5]. For this reason the proposed method could be conveniently used to perform Montecarlo procedures when uncertain parameters are considered, since the CPU time consumption of each run is lower if compared to standard techniques.

3 Response Bounds for MTL with Uncertain Parameters

In this section we define a technique for the evaluation of an upper and lower bound of the time domain response of a MTL when the per unit length parameters are uncertain. As stated in the introduction, this problem is of great importance, and nowadays the possibilities of dealing with it are somehow limited, mainly consisting of Montecarlo procedures and statistical models based on some simplifying approximations. The procedure we propose here is based on the model of the MTL obtained as recalled in the previous section, and as it will be shown, the computational cost is reasonably low.

3.1 Definition of the Equivalent Sources

We start this section by considering the transmission line represented by the algebraic system (1), obtained by performing the WE of the MTL equations. We underline here that the proposed technique can be applied to any other numerical model where the time variation of voltages and currents is expanded on a basis of functions, as it is for example done in [6]; for this reason we refer to the general algebraic system (5)

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{5}$$

where $\mathbf{x}$ is the vector of unknown coefficients of voltages and currents at the port of each cell, in which the line is segmented and $\mathbf{b}$ is a vector containing the input signal of the line.

In case of uncertainty of the line parameters, equation (5) can be seen as the representation of the MTL where the per unit length parameters assume the nominal value.

When a variation is considered, the new system can be written as

$$\tilde{\mathbf{A}}\tilde{\mathbf{x}} = \mathbf{b} \tag{6}$$

where $\tilde{\mathbf{A}}$ is the new matrix resulting from varied r, l, g, c; $\tilde{\mathbf{x}}$ is the new solution and $\mathbf{b}$ remains unchanged since the line is considered energized by the same signal. Equation (6) can be more conveniently written as

$$(\mathbf{A} + \mathbf{\Delta A})\tilde{\mathbf{x}} = (\mathbf{I} + \mathbf{\Delta A}\mathbf{A^{-1}})\mathbf{A}\tilde{\mathbf{x}} = \mathbf{b} \tag{7}$$

where the variation of the matrix $\mathbf{A}$ is now evidenced. The solution to equation (7) can be written as

$$\tilde{\mathbf{x}} = \mathbf{A}^{-1}(\mathbf{I} + \mathbf{\Delta A}\mathbf{A^{-1}})^{-1}\mathbf{b}. \tag{8}$$

By comparing equations (5) and (8) it is easy to see that

$$\tilde{\mathbf{b}} = (\mathbf{I} + \mathbf{\Delta A}\mathbf{A^{-1}})^{-1}\mathbf{b} \tag{9}$$

can be seen as a new input vector for the line with the nominal values of the parameters, and the varied transmission line response can be calculated by solving the system

$$\mathbf{A}\mathbf{x} = \tilde{\mathbf{b}}. \tag{10}$$

This means that the effect of the parameters variation has been moved from the system matrix to the vector of the input signals, changing it from $\mathbf{b}$ to $\tilde{\mathbf{b}}$.

As shown in [4] (and recalled in section II) the vector $\mathbf{b}$ contains the imposed voltages and currents at the ports of each cell; since the line can be considered as excited only at the terminals, its entries are all zeroes except than the bottom part (representing the Wavelet Expansion of the feeding generator). On the contrary the new vector $\tilde{\mathbf{b}}$ is in general a full vector, and by inverse transforming it we obtain the time domain behavior of the distributed sources that take into account the effects of the parameters variation, which are zero for the nominal line.

It can be easily seen that the way the time domain equivalent generators are obtained is easy and straightforward, and requires low CPU time (due to the wavelet properties): the total cost is an inversion of a sparse matrix (the "nominal" matrix, hence to be performed only once, even if several evaluations need to be performed) and a linear system solution. An analysis of the waveform and the magnitude of the generators allows us to obtain an insight in the effect of the parameters variation, and how it affects the output variation with respect to the nominal value.

Let us suppose that the parameters vary in a range expressed by $r = r_n \pm \Delta x_r\%$, $l = l_n \pm \Delta x_l\%$, $g = g_n \pm \Delta x_g\%$, and $c = c_n \pm x_c\%$, where the subscript n is related to the nominal value. It is possible to evaluate the distributed sources for the worst case condition, i.e. for $r = r_n(1 + \Delta x_r/100)$ and $r_n = r_n(1 - \Delta x_r/100)$, and so on. Performing (for a single conductor line) this operation for the whole set of parameters we have to perform a matrix inversion and solve eight linear systems. The result is the set of vectors $\tilde{\mathbf{b}}_{r-}$, $\tilde{\mathbf{b}}_{r+}$ $\tilde{\mathbf{b}}_{l-}$, $\tilde{\mathbf{b}}_{l+}$, $\tilde{\mathbf{b}}_{g-}$, $\tilde{\mathbf{b}}_{g+}$, $\tilde{\mathbf{b}}_{c-}$, $\tilde{\mathbf{b}}_{c+}$. For each of them it is possible to calculate the vector $\mathbf{b}' = \tilde{\mathbf{b}} - \mathbf{b}$, isolating the effects of the variation from the input generator.

3.2 Bounds Definition

The most common way to define the bounds of the response in presence of parameter's uncertainty is to perform a Montecarlo procedure, by repeating several simulations with a random variation of the parameters. An alternative approach has been studied by the authors in [8], where the bounds have been calculated by a first order approximation of the sensitivity with respect to the variables. Here we propose a different approach, based on the previous definition of the vectors $\tilde{\mathbf{b}}$ defined as follows.

Given a parameters' variation as expressed at the end of the previous section, we define the upper bound as related to a set of distributed sources constructed by adding up together the absolute values of the previously calculated vectors $\mathbf{b}'$, i.e.

$$\begin{aligned}\mathbf{b}_{upper} = |\mathbf{b}'_{r+}| + |\mathbf{b}'_{l+}| + |\mathbf{b}'_{g+}| + |\mathbf{b}'_{c+}| + \\ + |\mathbf{b}'_{r-}| + |\mathbf{b}'_{l-}| + |\mathbf{b}'_{g-}| + |\mathbf{b}'_{c-}| + \mathbf{b}.\end{aligned} \tag{11}$$

In this way it is possible to obtain the upper bound of the response straightforwardly resolving system (5) with the known term previously calculated, i.e.

$$\mathbf{x}_{upper} = \mathbf{A}^{-1}\mathbf{b}_{upper} \tag{12}$$

which can also be written as

$$\mathbf{x}_{upper} = \Delta\mathbf{x} + \mathbf{x}. \tag{13}$$

The lower bound of the response can now be calculated with no need of a further simulation, i.e.

$$\mathbf{x}_{lower} = \Delta\mathbf{x} - \mathbf{x}. \tag{14}$$

The two vectors $\mathbf{x}_{upper}$ and $\mathbf{x}_{lower}$ must be inverse transformed giving the time domain bounds.

At this point it is important once again to underline the computational cost of the whole procedure: with a line with N conductors, there are $NP = 4 * (N - 1)$ line parameters which are supposed to vary. By the use of the proposed method the computational cost is the following:

- a simulation with the nominal values, at the cost of an algebraic system solution;
- a matrix inversion;
- $2NP$ matrix-vector products, as in (9), obtaining the the new known vectors;
- absolute value operation and a sum, as in (11);
- a simulation from which we determine the response bounds.

It is hence evident the very low computational cost of the method if compared with a standard Montecarlo procedure. It can be interesting to analyze how wide can be the range of variation of the parameters that still permits a reasonable evaluation of the bounds. Some qualitative considerations can be made: the proposed procedure allows us to obtain an estimate of the response bounds in the presence of uncertainties; the simple evaluation reported in (11) is of course valid in a certain range of variation. In particular, we implicitly infere that the variation of the vectors $\tilde{\mathbf{b}}$ is monotonic in the range of variation of the parameters. Based on our experience the ranges of uncertainty in the parameters (due to aging or industrial tolerances) always satisfy the above mentioned requirement. It is noteworthy that a deeper analysis related to the wideness on the interval of uncertainty could be performed observing the magnitudes of the entries of matrix $\Delta\mathbf{A}$.

4 Numerical Results

In this section the results related to two different test cases are reported: first a simple bifilar line is considered, and a qualitative analysis of the equivalent sources, together with a comparison of the bounds obtained by the technique presented in [8] and by a Montecarlo procedure is shown. As a second test case a more complex 4 conductors line is chosen, showing the calculated bounds compared with a simulation technique.

4.1 Bifilar Line

The 2 conductors line is characterized by the following parameters:

$$\begin{aligned} r = 200\Omega/m, \qquad l = 2.8\mu H/m, \\ c = 1.2nF/m, \qquad g = 0S/m \end{aligned} \tag{15}$$

the line is characterized by a length of $L = 0.03$ m and is terminated at both ends on 50-Ω resistors. The feeding generator is a trapezoidal

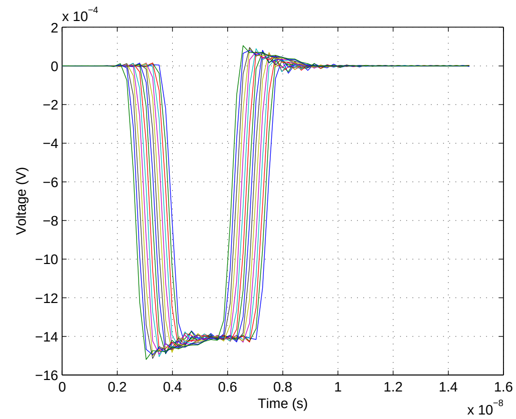

Figure 1: Distributed voltage sources related to the +10 % variation of r.

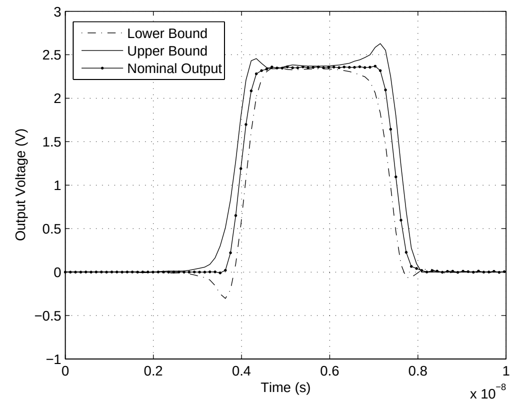

Figure 3: Calculated bounds and nominal output.

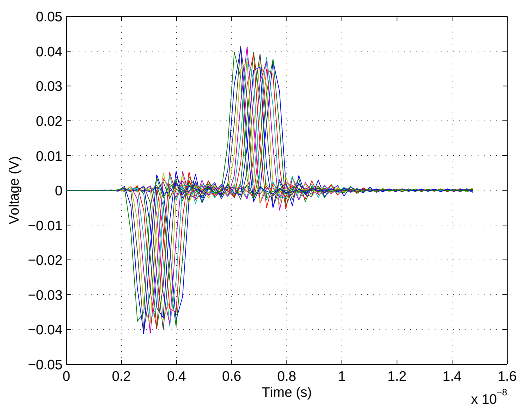

Figure 2: Distributed voltage sources related to the +10 % variation of l.

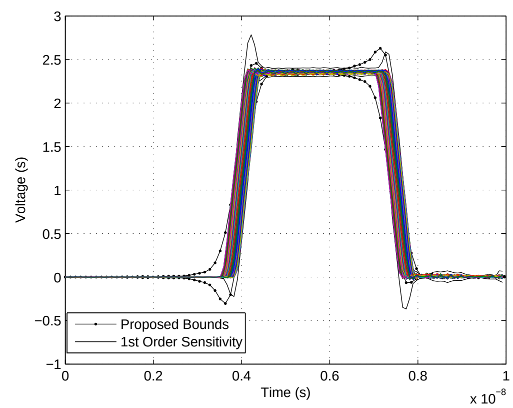

Figure 4: Calculated bounds and Montecarlo cloud.

pulse characterized by an amplitude of 5 V and a rise time of 0.5 ns. A variation of ±10% has been taken into account for all the parameters.

Figures 1 and 2 shows the time behavior of the distributed sources obtained considering a variation of resistance and inductance; analyzing the figure it is easy to verify the consistency of the obtained results and at the same time some conclusions can be drawn. In particular, we can see that the resistance variation mainly affects the steady state value of the voltages, while a variation of the inductance mainly influences the rise and fall time of the signal.

In a wider extent by performing a simple qualitative analysis on all the distributed sources related to all the parameters of a MTL it is possible to identify the most sensitive parameters.

Figure 3 shows the calculated bounds with respect to the result obtained by the nominal values of the parameters, while Fig. 4 shows a comparison between the bounds calculated by the proposed technique, with the bounds calculated as in [8] and with the Montecarlo cloud obtained performing 1000 random simulations.

Figures 3 and 4 show the accuracy of the method with respect to Montecarlo simulations.

4.2 Multiconductor Transmission Line

This second test case regards a 4 conductor transmission line, reported in Fig. 5

Figure 5: Multiconductor transmission line.

with the following parameters:

$$\mathbf{R} = \begin{vmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{vmatrix} \frac{\Omega}{m} \quad \mathbf{L} = \begin{vmatrix} 231 & 95 & 0 \\ 95 & 231 & 95 \\ 0 & 95 & 231 \end{vmatrix} \frac{nH}{m}$$

$$\mathbf{G} = \begin{vmatrix} 15.6 & 0 & 0 \\ 0 & 15.6 & 0 \\ 0 & 0 & 15.6 \end{vmatrix} \frac{\mu S}{m}$$

$$\mathbf{C} = \begin{vmatrix} 109 & -48 & 0 \\ -48 & 157 & -48 \\ 0 & -48 & 109 \end{vmatrix} \frac{pF}{m}$$

the line is characterized by a length of $L = 0.0156$ m while the values of the input and output resistances are: $R_1 = 30\ \Omega$, $R_2 = 10\ \text{M}\Omega$, $R_3 = 30\ \Omega$, $R_4 = 200\ \text{k}\Omega$, $R_5 = 60\ \Omega$, $R_6 = 50\ \text{k}\Omega$.

Figures 6, 7, and 8 show the calculated bounds at the far end of the line (i.e. the voltages at the resistances R_4, R_5, and R_6 and the respective Montecarlo cloud calculated with 5000 random variation. The accuracy of the calculated bound can be easily seen in the figures.

A general comment can be made on the CPU time cost of the proposed method: as clearly adressed in section 3 the most relevant time consuming the activity is the inversion of the matrix **A**, clearly depending on its dimension. The dimension of the above mentioned matrix depends on the frequency content of the input signal and the number of cells in which the line must be divided (more details can be found in [4]); these two parameters are related to the velocity of propagation of the signal along the line and the line length itself. The examples chosen contain typical signals and line length characteristics of high speed interconnects. In order

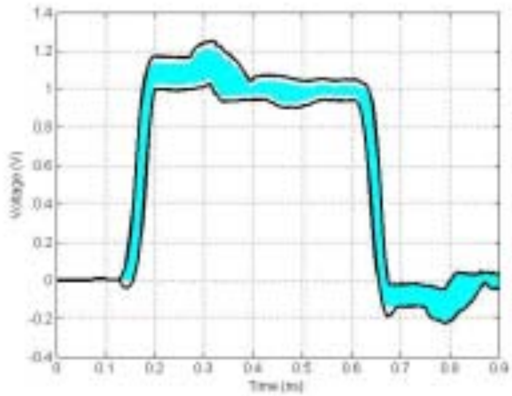

Figure 6: Calculated bounds and Montecarlo cloud for conductor 1.

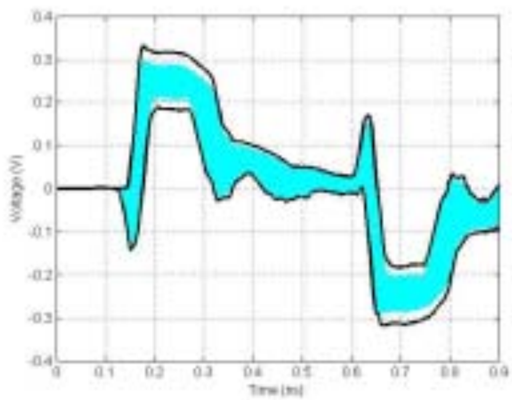

Figure 7: Calculated bounds and Montecarlo cloud for conductor 2.

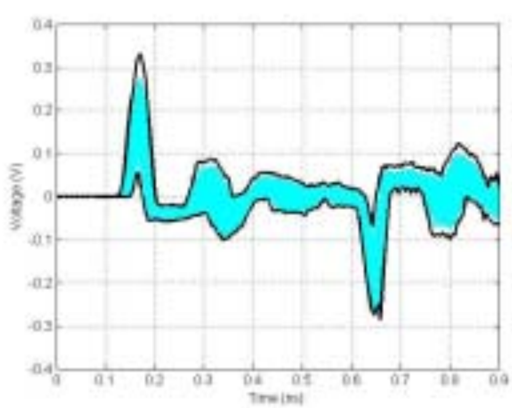

Figure 8: Calculated bounds and Montecarlo cloud for conductor 3.

to compare the proposed method and a Montecarlo procedure, it is necessary to remind an important difference: by the use of the proposed method the number of calculations (henceforth the CPU time) is determined, and allows us to find directly the bounds. As a matter of fact there is no apriori knowledge on the number of Montecarlo runs necessary to obtain a reasonable upper and lower limit, therefore the total number is chosen based on qualitative considerations. The comparison we have performed regard a number of Montecarlo simulations creating a cloud which is almost unchanged if we add another set of simulations. Under this assumption the proposed method is characterized by lowering the necessary CPU time of an order of magnitude and more.

5 Conclusion

In this paper a method for the analysis and evaluation of the response bounds of a Transmission Line characterized by uncertain parameters is shown. The bounds so obtained are compared with a Montecarlo simulation, showing the significance of the result.

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References

[1] S. Pignari and D. Bellan "Statistical Characterization of Multiconductor Transmission Lines Illuminated by a Random Plane - Wave Field," Proceedings of IEEE International Symposium on Electromagnetic Compatibility, 2000, vol. 2, pp. 605 - 609, 21-25 Aug. 2000.

[2] S. Shiran, B. Reiser, and H. Cory "A Probabilistic Model for the Evaluation of Coupling Between Transmission Lines," IEEE Trans. Electromagnetic Compatibility, vol. 35, pp. 387-393, Aug. 1993.

[3] S. Lun, M. S. Nakhla, and Q-J Zhang, "Sensitivity Analysis of Lossy Coupled Transmission Lines," IEEE Transaction on Microwave Theory and Techniques, vol. 39, n. 12, pp. 2089 - 2099, December 1991.

[4] S. Barmada and M. Raugi, "Transient numerical solution of nonuniform MTL equations with nonlinear loads by wavelet expansion in time or space domain," IEEE Trans. on Circuits and Systems, vol. 47, Issue 8, pp. 1178 - 1190, August 2000.

[5] S. Barmada, A. Musolino, and M. Raugi "Wavelet Based Time Domain Solution of Multiconductor Transmission Lines with Skin and Proximity Effect," accepted for publication on IEEE Trans. EMC.

[6] Y. S. Chung, T. K. Sarkar, B. H. Jung, M. Salazar - Palma, Z. Ji, S. Jang, and K. Kim, "Solution of Time Domain Electric Field Integral Equation Using the Laguerre Polynomials," IEEE Transaction on Antennas and Propagation, vol. 52, no. 9, pp. 2319 - 2328, September 2004.

[7] G. Costache, "Transient Proximity Effect and Longitudinal Transient Parameters of a Bifilar Transmission Line," Rev. Roum. Sci. Techn. - Electrotechn. et Energ., vol. 16, no. 1, pp. 111-124, 1971.

[8] S. Barmada, G. Becherini, A. Musolino, and M. Raugi, "A Sensitivity Approach to Determine the Response Bounds of Transmission Lines with Random Parameters," Proceedings of the 2004 IEEE International Symposium on Electromagnetic Compatibility, Santa Clara, USA, pp. 413 - 418, 9 - 13 August 2004.

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