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Atef Z. Elsherbeni

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# THE APPLIED COMPUTATIONAL ELECTROMAGNETICS SOCIETY <br> JOURNAL 

Vol. 21 No. 2
July 2006

## TABLE OF CONTENTS

"The Spherical Harmonic Interface Procedure for MM and UTD Codes"
J. Rockway, E. Newman and R. Marhefka ..... 105
"Mutual Coupling between Monopoles on F-4 Aircraft at Transition Frequencies: A Comparison between MoM and MoM/UTD Hybrid Method at UHF Band"
A. Dagdeviren, O. Cerezci, F. Ustuner, and B. Turetken ..... 111
"A New Iterative Method to Compute the Higher Order Contributions to the Scattered Field by Complex Structures"
F. Saez de Adana, O. Gutiérrez, L. Lozano, and M. Cátedra ..... 115
"The Use of Multiquadric Radial Basis Functions in Open Region Problems"
R. Gordon and W. Hutchcraft ..... 127
"A Novel Dual-band Small Size Microstrip Antenna"
A. Sheta, H. Boghdady, A. Mohra, and S. Mahmoud ..... 135
"An Efficient Broadband Analysis of an Antenna via 3D FEM and Pade Approximation in the Frequency Domain"
B. Essakhi and L. Pichon ..... 143
"EBG Design using FSS Elements in Rectangular Waveguide"
R. Kshetrimayum and L. Zhu. ..... 149
"Electromagnetic Modeling and Design of Dual-Band Septum Polarizers"
A. Kirilenko, D. Kulik, L. Rud, V. Tkachenko, and N. Herscovici ..... 155
"The Influence of Data Density on the Consistency of Performance of the Feature Selective Validation (FSV) Technique"
A. Duffy and A. Orlandi ..... 164
"To-Average or Not-to-Average in FDTD Modeling of Dielectric Interfaces" C. Furse, C. Waterman, and L. Griffiths ..... 173

# The Spherical Harmonic Interface Procedure for MM and UTD Codes 

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#### Abstract

This paper describes the use of a spherical harmonic expansion as an efficient interface between a low frequency method of moments (MM) code and a high frequency uniform geometrical theory of diffraction (UTD) code. It is shown that the method can save significant CPU time in the UTD code provided that the number of MM current filaments per cubic wavelength is large.


## I. Introduction

In computational electromagnetics, when a problem involves both an electrically small part and an electrically large part, the standard approach is a hybrid solution [7, 12, 5, 11]. For example, consider the problem of an antenna radiating in the presence of an electrically large structure such as a ship or a building at X band. Typically, a low frequency method of moments (MM) code is used to find the currents on the antenna in free space, or in the presence of a small part of the structure closest to the antenna. These antenna currents are then input to a high frequency uniform geometrical theory of diffraction (UTD) code which determines their fields in the presence of the electrically large support structure.

The purpose of this paper is to describe the spherical harmonic interface procedure (SHIP) [9] in which a spherical harmonic expansion [10] of the free space antenna fields is used as an interface to the UTD code, rather than the MM antenna currents. The primary advantage of the SHIP is a result of the fact that in UTD ray tracing codes, the CPU time is proportional to the number of ray origins from which rays must be traced. Since for a complex antenna or large array, there could be thousands of MM currents, the UTD code would have to trace rays from thousands of origins. By contrast, a spherical harmonic expansion of the antenna fields has a single origin, and thus the UTD code need only trace rays from a single point. The CPU time for the SHIP is proportional to the number of spherical harmonics needed to accurately represent the antenna
fields, which in turn is proportional to the electrical size of the antenna.

A secondary advantage of the SHIP is that the UTD code can be written independent of the details of the low frequency code. The low frequency code can be based upon the MM, FEM, FDTD, etc., and can employ any basis functions. Providing that the UTD code is given the coefficients in the spherical harmonic expansion of the antenna fields, these details are not relevant to the UTD code.

Section II will begin with a description of the basic single origin or cell SHIP. It is then pointed out that there are two instances when it is either necessary or desirable to employ a multiple origin or multiple cell SHIP. First, the use of the spherical harmonic expansion requires the SHIP cells to be in the far zone of points of diffraction on the scattering structure. If an electrically large SHIP cell is in the near zone of the scatterer, then it must be split into smaller cells so that the points of diffraction on the scattering body are in the far zone of each cell. Second, it will be shown that it is possible to reduce the CPU time of the UTD code by employing a multiple cell SHIP. The paper will conclude with examples illustrating the reduced CPU time for the SHIP versus the standard MM current approach. For all examples, the MM code will be The Electromagnetic Surface Patch Code: Version V (ESP5) [8], and the UTD code will be the NEC Basic Scattering Code (NEC-BSC) [6]. All UTD code CPU times will be for a far zone pattern at 360 angles. All CPU times are for a PC with a $1.7 \mathrm{GHz} \operatorname{Intel}(\mathrm{T})$ Pentium M processor.

## II. Description of the SHIP

## II. A. Single Cell SHIP

A hybrid MM/UTD solution for an antenna radiating in the presence of a large structure begins with the MM code determining the current on the antenna,
and then the UTD code finds the fields produced by these currents in the presence of the large structure. Rather than have the UTD code trace rays from each MM current segment or current filament, one can construct a spherical wave expansion of the fields of the free space fields of the MM currents. In the far zone of a sphere of radius $r_{1}$ enclosing the currents, the spherical wave expansion for each component of the electric field is of the form

$$
\begin{equation*}
E(r, \theta, \varphi)=A \sum_{m=0}^{N_{\text {max }}} \sum_{n=m}^{N_{\text {ma1 }}}\left[a_{m n} Y_{m n}^{e}+b_{m n} Y_{m n}^{o}\right] \frac{e^{-j k r}}{r} . \tag{1}
\end{equation*}
$$

For simplicity we have assumed the enclosing sphere is centered at the coordinate origin, the subscript 1 emphasizes that this is for a single origin expansion, and the reader is referred to [1] for a detailed description of the various terms. To obtain reasonable accuracy from equation (1), the number of terms which must be kept in the summations is approximately [2-4]

$$
\begin{equation*}
N_{\max 1}=k r_{1}+3 \ln \left(k r_{1}+\pi\right) . \tag{2}
\end{equation*}
$$

In the SHIP, the MM code provides the UTD code with the $a_{m n}$ and $b_{m n}$ coefficients of equation (1), and thus the UTD code need only trace rays from a single coordinate origin. It is also is completely divorced from the details of the MM (or other) solution. The CPU time of the UTD code will be dependent upon the number of harmonics which must be summed in equation(1),

$$
\begin{equation*}
N_{H 1}=N_{\max 1}^{2} \approx\left(k r_{1}\right)^{2} \quad \text { if } k r_{1} \gg 1 . \tag{3}
\end{equation*}
$$

For the NEC-BSC code, Figure 1 shows a log-log plot of the CPU time versus $N_{H 1}$ for the scattering body being free space (i.e., no scatterer) and for the wedge of Figure 2. Free space and the wedge represent extremes in terms of the complexity of the ray trace. Noting that for large $N_{H 1}$, these two extremes produce straight lines with essentially the same slope, the single cell CPU time must be of the form

$$
\begin{equation*}
T_{1}=C N_{H 1}^{\beta}=C\left(k r_{1}\right)^{2 \beta} \tag{4}
\end{equation*}
$$

where from the slope of the lines $\beta \approx 1.45 . C$ is a constant dependent upon the complexity of the ray trace, and a reasonable fit to the data of Figure 1 is

$$
\begin{equation*}
C_{F S p} \approx 6.82 \mu \mathrm{sec}, \quad C_{\text {Wedge }} \approx 85.5 \mu \mathrm{sec} . \tag{5}
\end{equation*}
$$

Note that one should always choose the coordinate origin for the spherical wave expansion near the center of the MM currents, since this will minimize $r_{1}$ and thus the number of required harmonics, $N_{H 1}$.

## II. B. Multiple Cell SHIP

By a $P$ cell SHIP it is meant that the single cell of radius $r_{1}$ is segmented into $P$ smaller cells, and the


Fig. 1. CPU times for the single cell SHIP in free space and for the wedge.


Fig. 2. Geometry for a PEC square base wedge.
total fields are expressed as the sum of the spherical wave expansions for the fields of the currents located in each of the $P$ cells. If the single cell of radius $r_{1}$ is not in the far zone of a point of diffraction on the scatterer, then it must be segmented into smaller cells which are in the far zone. The cells may be in the near zone of a point of reflection since the NEC-BSC (and we assume most UTD codes) treat this via image theory. The $P$ cell SHIP tends to increase the CPU time since the UTD code must trace rays from $P$ origins, however, each ray trace is faster since the cells are smaller and thus require fewer harmonics. As shown below, it is possible to
reduce the overall UTD code CPU time by employing a multiple cell SHIP.

Referring to Figure 3, for simplicity we will assume that the radiating body can be classified, based upon its overall dimensions, as one of the following:

- 1D Body: 1 large and 2 small dimensions (linear array),
- 2D Body: 2 large and 1 small dimensions (square array),
- 3D Body: 3 large dimensions (cubic array).

In this case, the cell radius for $P$ cells is simply related to that for the single cell by

$$
\begin{equation*}
r_{P}=\frac{r_{1}}{P^{\alpha}} \tag{6}
\end{equation*}
$$

where $\alpha_{1 D}=1, \alpha_{2 D}=1 / 2$, and $\alpha_{3 D}=1 / 3$ for the three cases.
-The number of harmonics in each of the $P$ cells is

$$
\begin{equation*}
N_{H P}=N_{\max P}^{2}=\left(k r_{P}\right)^{2}=\left(k \frac{r_{1}}{P^{\alpha}}\right)^{2}=\frac{N_{H 1}}{P^{2 \alpha}} . \tag{7}
\end{equation*}
$$

Since the UTD code must trace rays from each of the $P$ origins, the total CPU time for the $P$ cell SHIP is

$$
\begin{equation*}
T_{P}=P\left[C N_{H P}^{\beta}\right]=P C\left(\frac{N_{H 1}}{P^{2 \alpha}}\right)^{\beta}=P^{1-2 \alpha \beta} T_{1}=F_{P} T_{1} \tag{8}
\end{equation*}
$$

where

$$
\begin{equation*}
F_{P}=P^{1-2 \alpha \beta} \tag{9}
\end{equation*}
$$

is the factor to convert the single cell to the $P$ cell CPU time. For the three cases above

- 1D Body: $F_{P}=P^{1-2(1)(1.45)}=P^{-1.9}$,
- 2D Body: $F_{P}=P^{1-2(1 / 2)(1.45)}=P^{-0.45}$,
- 3D Body: $F_{P}=P^{1-2(1 / 3)(1.45)}=P^{+0.033}$.

It follows that for 1D and 2D bodies, one should use as many cells as possible, while for 3D bodies there is always a slight disadvantage to segmentation.

There are three important caveats to the above statement. First, one must always segment so that the SHIP cells are small enough that they are in the far zone of the points of diffraction on the scattering body. Second, equation (8) will accurately predict $T_{P}$ only if $N_{H P}=N_{H 1} / P^{2 \alpha}$ is large enough to be on the linear portion of Figure 1. For smaller values of $N_{H P}$, equation (8) should be considered as qualitative only. This problem could be removed by making the slope, $\beta$ a function of the number of the number of harmonics in a single cell; however, for simplicity it was not done here. Finally, it is assumed that as the radiating body is segmented, each smaller cell contains at least 1 MM current, so that rays must be traced from each SHIP cell.

(a) A 1D shape is segmented into $P=6$ cells.

(b) A 2D shape is segmented into $P=16$ cells.

(c) A 3D shape is segmented into $P=64$ cells.

Fig. 2. A 1D, 2D, and 3D source is segmented in $P$ cells.
For a 1 D body radiating in the presence of the wedge, Figure 4 shows NEC-BSC CPU times for a $P=$ 1,2 , and 4 cell SHIP versus $N_{H 1}=$ the number of single cell harmonics. According to equation (7), the number


Fig. 3. CPU times for the 1D P cell SHIP in the presence of the wedge.
of harmonics in each cell of the $P$ cell SHIP is $N_{H 2}=N_{H 1} / 4$ and $N_{H 4}=N_{H 1} / 16$. The solid lines are the actual NEC-BSC times, while the dashed lines are obtained by applying the $F_{P}$ factor of equation (9). The reason that the approximation for the $P=2$ curve is better than that for $P=4$, is that $N_{H 2}=4 N_{H 4}$, and thus is more on the linear portion of the curves in Figure 1.

## II. C. SHIP vs. Standard MM Current CPU Times

The CPU time for the standard approach, in which the UTD code superimposes the field of each MM current filament, is $T_{M M F}=D N_{F}$, where $N_{F}$ is the number of MM filaments, and $D$ is a constant dependent upon the complexity of the ray trace. For the NEC-BSC code, and the wedge of Figure 2,

$$
\begin{equation*}
D_{\text {Wedge }} \approx 87.8 \mathrm{msec} . \tag{10}
\end{equation*}
$$

For the wedge, the ratio of the CPU time for the standard MM filament approach to that for the $P$ cell SHIP is

$$
\begin{equation*}
R=\frac{T_{M M F}}{T_{P}}=\frac{D_{\text {Wedge }} N_{F}}{F_{P}\left(C_{\text {Wedge }} N_{H 1}^{\beta}\right)} \approx \frac{5}{F_{P}} \frac{N_{F}}{\left(r_{1} / \lambda\right)^{2.9}} . \tag{11}
\end{equation*}
$$

This indicates that the $P$ cell SHIP will be faster than the standard filament approach if

$$
\begin{equation*}
\frac{N_{F}}{F_{P}\left(r_{1} / \lambda\right)^{2.9}} \approx \frac{N_{F}}{F_{P}\left(r_{1} / \lambda\right)^{3}}>0.2 \tag{12}
\end{equation*}
$$

i.e., if the number of MM filaments per cubic wavelength is large.

## II. D. SHIP Examples

This section will present two examples illustrating the benefits of the SHIP. The first example will be a $50 \lambda \times 50 \lambda$ square array of dipoles over the PEC wedge of Figure 2. By making a series of runs with increasing density of dipoles within the fixed $50 \lambda \times 50 \lambda$ square, Figure 5 shows the UTD code CPU time versus the number of MM filaments. Since the electrical size of the source is fixed, the CPU times for the SHIP are independent of the number of MM filaments. Note that the $P=1$ or 4 cell SHIP is faster than the standard filament approach when $N_{f}>6500$ or 4200 filaments, respectively. Using $r_{1}=25 \sqrt{2} \lambda$, equation (12) predicts that the $P=1$ or 4 cell 2D SHIP will be faster than the standard filament approach when $N_{f}>6200$ or 3300 filaments, respectively. The decrease in accuracy for the $P=4$ cell SHIP is a result of not being on the linear portion of Figure 1.


Fig. 4. UTD CPU times for a dipole array above the wedge.

Figure 6 shows a UTD plate model of a ship with a $30 \times 30$ array of $\lambda / 2$ dipoles in a $15 \lambda \times 15 \lambda$ square. Using the standard filament approach, the UTD CPU time to compute an azimuth pattern was 351 min., but required only 261 sec . for a $P=16$ cell 2D SHIP. This represents a reduction in CPU time by a factor of about 80. The CPU time to compute the $a_{m n}$ and $b_{m n}$ coefficients of equation (1) was only 61 sec .


Fig. 5. A UTD model of a ship with a $30 \times 30$ array of dipoles in a $15 \lambda \times 15 \lambda$ square.

## III. SUMMARY

This paper has described the use of a spherical harmonic expansion to reduce the UTD code CPU times in a hybrid MM/UTD solution. The advantage of the SHIP is that it reduces the number of origins from which the UTD code must trace rays. The SHIP is shown to be effective in reducing the UTD code CPU time when the number of MM filaments per cubic wavelength is large.

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# Mutual Coupling between Monopoles on F-4 Aircraft at Transition Frequencies: A Comparison between MoM and MoM/UTD Hybrid Method at UHF Band 

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#### Abstract

In this paper, the analysis of mutual coupling between monopole antennas mounted on the aircraft at UHF band is presented. The Method of Moments (MoM) and hybridized MoM with Uniform Geometrical Theory of Diffraction (UTD) method (MoM/UTD) are used to calculate the coupling in 225 $\mathrm{MHz}-400 \mathrm{MHz}$ frequency range. Numerical results with MoM and $\mathrm{MoM} / \mathrm{UTD}$ are performed and the reliability of these methods is discussed based on the measurement results.


Keywords - Antenna Coupling, MoM, MoM/UTD

## I. Introduction

Aircrafts have many communication and navigation systems for different purposes. The radiated electromagnetic interference (EMI) between the antennas of these systems is a potential serious problem and always has to be considered. The power output of the transmitter systems, sensitivity of the receiver systems and the coupling between the antennas mainly determine the EMI safety range [1,2]. So one has to compute or measure the coupling between two antennas to decide whether a potential EMI risk exists. The analytical calculation of the coupling between surface mounted antennas on aircraft or similar complex structures is difficult and time consuming, so predictions by either numerical or asymptotic methods are preferred [3].
The electrical size of the structure to be analyzed increases at high frequencies because of the shorter wavelength. Practically, these methods can be used up to UHF band. High frequency asymptotic methods, like UTD, should be utilized for electrically large structures. However, this method also has some limitations like others. For low frequencies $\mathrm{kL}>1$ limitation should be considered [4]. Here $k$ is the wavenumber and $L$ stands for the distance parameters as defined in [4]. Generally, this method becomes practical beginning from UHF. At high frequencies, the accuracy of the model becomes a limitation [5]. So, UHF band can be considered as a transition frequency range from low frequency
methods to high frequency methods. A comparison between MoM and MoM/UTD method at transition frequencies is presented.

## II. ANALYSIS

The analysis has been performed in the 225 MHz 400 MHz frequency range between COM2/V-UHF radio communication antennas located on the top and the bottom of the fuselage and on the tail (Fig. 1). The antennas are quarter wavelength monopoles. The simulations are realized by two different methods: MoM and MoM/UTD Hybrid Method.
Two scenarios have been investigated in this work: The mutual coupling between A1-A2 and A2-A3. The location of the antennas on the aircraft can be seen in Fig. 1.


Figure 1. MoM Model and Antenna Locations.
Segment and grid sizes of around $1 / 10$ wavelength or less. The problem solution time is proportional to the cube of the number of segments while the computermemory requirement is proportional to the square of the number of segments. Here, 12 cross sectional data of the model are defined, and subsequently models are generated for 175 MHz . The segment length is chosen to be equal to $\lambda / 10$ at the frequency of interest. The total number of segments for the aircraft adds up to 10910.

High modeling frequency, so lower wavelength and segment length, increases the segment number of the model. For such a huge amount of segments, MoM analysis requires modeling the structure surfaces using wire segments rather than using patch model of surfaces.
Since the size of the aircraft becomes significant in terms of wavelength at UHF and above, the low frequency methods like MoM are not useful due to
long computation time and high memory requirement at these frequencies. So, an asymptotic method, the UTD is normally used. This technique based on ray tracing, considers the propagation of Electromagnetic (EM) waves as tubes of rays [6]. At each point in the space, the total EM field is calculated by superposing the incident field, reflected fields, diffracted fields and all other higher order interactions (double reflection, diffraction-reflection, reflection-diffraction etc.).
In the MoM/UTD Hybrid Method, the MoM interaction Matrix is modified according to the interaction between wire segments via UTD objects: flat plates and cylinders. The MoM/UTD hybrid model of the aircraft consists of 18 plates, 9 cylinders and wire-segmented monopoles.
The technique used to hybridize the MoM and UTD was presented by Thiele and Newhouse [7-8]. Basically, the original MoM matrix elements are increased with the additional fields due to interaction mechanisms up to third order.
In short, the $m n^{\prime}$ th element of the new impedance matrix is

$$
\begin{equation*}
\mathrm{Z}_{\mathrm{mn}}^{\prime}=\mathrm{Z}_{\mathrm{mn}}+\mathrm{Z}_{\mathrm{mn}}^{\mathrm{g}} \tag{1}
\end{equation*}
$$

Here $Z_{m n}$ is due to direct field from source to observation point and $Z_{m n}^{g}$ is due to other interactions (reflection, diffraction...).

The Linville method, a technique used in RF amplifier design, is used to compute the maximum coupling between the two antennas. Then mismatch losses are considered to reduce to maximum coupling due to input and output load mismatches. The maximum coupling according to Linville method is [9]:

$$
\begin{equation*}
\mathrm{C}_{\max }=\frac{1-\sqrt{1-\mathrm{L}^{2}}}{\mathrm{~L}} \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{L}=\frac{\left|\mathrm{Y}_{12} \mathrm{Y}_{21}\right|}{2 \operatorname{Re}\left[\mathrm{Y}_{11}\right] \cdot \operatorname{Re}\left[\mathrm{Y}_{22}\right]-\operatorname{Re}\left[\mathrm{Y}_{12} \mathrm{Y}_{21}\right]} . \tag{3}
\end{equation*}
$$

As can be seen in the equation above, to calculate the maximum coupling given in eq.(2), firstly the twoport admittance parameters for the coupled antennas should be determined by exciting each antenna with the other short-circuited, then the self and mutual admittance from the currents computed by NEC should be computed [9].

In case of maximum coupling load admittance on antenna 2 must be matched[10]:

$$
\begin{equation*}
Y_{L}=\left[\frac{1-\rho}{1+\rho}+1\right] \operatorname{Re}\left[Y_{22}\right]-Y_{22} \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\rho=\frac{\mathrm{C}_{\max }\left(\mathrm{Y}_{12} \mathrm{Y}_{21}\right)^{*}}{\left|\mathrm{Y}_{12} \mathrm{Y}_{21}\right|} \tag{5}
\end{equation*}
$$

and the corresponding input admittance of antenna 1 is:

$$
\begin{equation*}
Y_{\mathrm{IN}}=Y_{11}-\frac{Y_{21} Y_{12}}{Y_{L}+Y_{22}} \tag{6}
\end{equation*}
$$

The mismatch losses are calculated according to:

$$
\begin{align*}
& \text { MLin }=10 \cdot \log \left(1-\left|\Gamma_{\text {in }}\right|^{2}\right)^{0} \\
& \text { MLout }=10 \cdot \log \left(1-\left|\Gamma_{\text {out }}\right|^{2}\right) \tag{7}
\end{align*}
$$

where

$$
\begin{align*}
& \Gamma_{i n}=\frac{Z_{i n}-Z_{o}}{Z_{i n}+Z_{o}} \\
& \Gamma_{o u t}=\frac{Z_{L}^{*}-Z_{o}}{Z_{L}^{*}+Z_{o}} \tag{8}
\end{align*}
$$

## III. Measurement and Results

Antenna coupling measurements have been performed in an $8.4 \mathrm{~m} \times 4.8 \mathrm{~m} \times 3.3 \mathrm{~m}$ Semi-Anechoic Chamber (SAC). A 1:10 scaled model of the F-4 aircraft was used. The measurement setup picture can be found in Fig. 2 and depicted in Fig. 3.


Figure 2. Coupling Measurements in SAC.


Figure 3. Detailed Measurement Setup Drawing in SAC.

The inner dimensions of the room are approximately 7.2 m long, 3.6 m wide, and 2.7 m high from the tip of the absorbers.
The frequency range to be considered for the analysis is $225 \mathrm{MHz}-400 \mathrm{MHz}$. The aircraft model has $1: 10$ scaling, so the measurement was performed in 2.25
$\mathrm{GHz}-4 \mathrm{GHz}$ frequency range. $\mathrm{S}_{21}, \mathrm{~S}_{11}, \mathrm{~S}_{22}$ parameters were measured via the network analyzer, then mismatch losses and optimum coupling were calculated according to these scattering parameters:

$$
\begin{gather*}
\mathrm{ML}_{1}=10 \cdot \log \left(1-\left|\mathrm{S}_{11}\right|^{2}\right)  \tag{9}\\
\mathrm{ML}_{2}=10 \cdot \log \left(1-\left|\mathrm{S}_{22}\right|^{2}\right)  \tag{10}\\
\mathrm{C}=10 \cdot \log \left(\left|\mathrm{~S}_{21}\right|^{2}\right)-\mathrm{ML}_{1}-\mathrm{ML}_{2} \tag{11}
\end{gather*}
$$

The MoM coupling and MoM/UTD Hybrid method coupling between three antennas on aircraft were computed, then measured in SAC and plotted (See Fig. 4. and Fig. 5).


Figure 4. Coupling Between A1 and A2.


Figure 5. Coupling Between A2 and A3.

## IV. Conclusion

In this work an efficient approach to the numerical analysis of mutual coupling between the UHF communication antennas of F-4 aircraft has been presented. The results show that coupling gathered from both methods approximated to measurement results nearly by $\pm 10 \mathrm{~dB}$. MoM results are closer to the measurement results with respect to MoM/UTD results.
The accuracy of the simulation results depends on how precisely the model represents the aircraft. For the measurements, the precision of the scaled model
used in the measurements influences the measurement accuracy. In the case of MoM/UTD, the number of objects used and the number of maximum interactions allowed by the software affects the simulation results. In the case of MoM , wire grid size is important for the accuracy of the results. Detailed information can be found in [11] and [12].
The authors emphasize that one will better use MoM model even at the transition frequencies. A quick analysis for simplified MoM/UTD model of the complex structure can be carried out for introductory information and at higher frequencies it will be mandatory to use this model.

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# A New Iterative Method to Compute the Higher Order Contributions to the Scattered Field by Complex Structures 

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#### Abstract

A method to compute the higher order contributions to the scattered field by complex structures is presented in this paper. The method is based on a new interpolation technique to represent the induced current with a very low amount of sample points and computational cost. The amplitude and phase of the current are represented separately. Both are defined by an interpolating function, which is built using Bézier surfaces. These functions provide the amplitude and the phase at any given point of the scattering surface in an easy way. The higher order contributions to the scattered field are obtained by using a new iterative method based on Physical Optics (PO) and the Stationary Phase Method (SPM) to compute the integral. The proposed method takes advantage of the saving in computation cost offered by the new representation of the currents reducing the order of the function which is necessary to minimize, in order to obtain the stationary phase points required to evaluate the PO integral. The results obtained show that the method is both efficient and accurate.


## I. INTRODUCTION

The main contributions to the scattered field by a simple object in presence of an electromagnetic wave are mainly due to the first order effects (reflections or diffractions). However, if the complexity of the body increases (for example, an airplane, a satellite or a ship), then higher order contributions (doublereflections, diffraction-reflections, etc) become relevant, especially for certain angular margins. There are many other situations where higher order effects are relevant: the analysis of antennas onboard complex structures, the study of propagation in tunnels, the computation of the RCS of cavities, etc. In these cases, multiple reflections and /or diffractions generally, make the greatest contribution to the scattered field.

Traditionally, there are some approximate methods to tackle the problem of high order interactions for asymptotically high frequencies. The Image Method (IM) [1-3] obtains multiple reflections by repeatedly applying the Image Theory [4], calculating multiple images of the electromagnetic
source and from these the scattered field at the observation point. The main problem with this technique is that the reflecting surfaces must be flat. Another method is the Shooting and Bouncing Rays (SBR) approach, [5-8], in which tubes of rays are shot from the source in all directions. When the tube reaches the observation point after reflecting in the body surface, the previously computed field intensity level at that point is amended with the tube's contribution to the field level. The main problem with this method is the high number of rays that must be shot to obtain the scattered field accurately enough, making the computational cost very high. As it is stated in [9] an exceedingly large number of rays must be traced for very high frequencies (sometimes up to 350 points per square wavelength). Another important problem is the treatment of the diffraction due to the high number of tubes of rays in the Keller's cone produced when an incident tube of rays reaches an edge.

Other possibilities are the inverse methods based on the Geometrical Theory of Diffraction (GTD/UTD) [10,11] or the SPM [12], in which, given the structure, the source and the observation points, all the possible reflection and diffraction paths connecting the source with the observation points are obtained, taking into consideration the contribution of certain flash-points to the scattered field: reflection points in GTD/UTD or stationary phase points in SPM. The main problem with these methods is obtaining the flash-points on the surfaces involved.


Fig. 1. Path to minimize for n reflections.

For example, Figure 1 shows a situation where a source and an observation point are placed in the vicinity of $n$ arbitrary surfaces. If one wants to compute the reflected field at the observation point due to an $n$-order reflection in the surfaces of the scene, the first thing to do, using inverse methods, is to obtain the position of the n reflection points. If the geometrical surfaces are represented by parametrical surfaces (NURBS or Bézier surfaces $[13,14]$ ) as it is stated in [10] and [12], the reflection points are obtained after the minimization of the following function:

$$
\begin{align*}
\mathrm{d}^{\mathrm{n}}\left(\mathrm{u}_{1}, \mathrm{v}_{1},\right. & \left.\mathrm{u}_{2}, \mathrm{v}_{2}, \ldots ., \mathrm{u}_{\mathrm{n}-1}, \mathrm{v}_{\mathrm{n}-1}, \mathrm{u}_{\mathrm{n}}, \mathrm{v}_{\mathrm{n}}\right) \\
& =\mathrm{d}_{0}\left(\mathrm{u}_{1}, \mathrm{v}_{1}\right)+\mathrm{d}_{1}\left(\mathrm{u}_{1}, \mathrm{v}_{1}, \mathrm{u}_{2}, \mathrm{v}_{2}\right)+ \\
& +\mathrm{d}_{2}\left(\mathrm{u}_{2}, \mathrm{v}_{2}, \mathrm{u}_{3}, \mathrm{v}_{3}\right)+\ldots .+ \\
& +\mathrm{d}_{\mathrm{n}-1}\left(\mathrm{u}_{\mathrm{n}-1}, \mathrm{v}_{\mathrm{n}-1}, \mathrm{u}_{\mathrm{n}}, \mathrm{v}_{\mathrm{n}}\right)+\mathrm{d}_{\mathrm{n}}\left(\mathrm{u}_{\mathrm{n}}, \mathrm{v}_{\mathrm{n}}\right) \tag{1}
\end{align*}
$$

where $\mathrm{d}^{\mathrm{n}}$ is the total distance of the ray path followed by the n-reflection, $d_{i}$ the different stretches in which $\mathrm{d}^{\mathrm{n}}$ can be divided (Figure 1) and $\mathrm{u}_{\mathrm{i}}, \mathrm{v}_{\mathrm{i}}$ the parametrical coordinates that define the surface i (see references [12-14]). As can be seen, it is necessary to minimize a function of 2 n variables. The problem is that the cost of this minimization increases exponentially with n .

The objective of the method proposed in this paper is to analyze electrically large bodies with multiple reflections and diffractions between its parts by computing efficiently the PO integrals using the SPM without the cumbersome and time-consuming problems of minimizing a function of $n$ variables as demand the inverse ray tracing methods or the need to shouting a huge number of ray tubes as requires the SBR method. The proposed method combines the interpolation of the induced current by means of the current modes proposed in [15] with the SPM [16, 17] to obtain the scattered field. As stated in [15] the current is interpolated by means of a Bezier surface from the induced currents on a set of sampling points over the surface. Given the behavior of Bezier surfaces $8 \times 8$ sampling points are a number optimum for a good representation of the current. After the interpolation the critical points of SPM can be obtained minimizing the phase function of the PO integrand that includes the phase of the induced current. As it will be described below, there is no relationship between the SPM critical points and the control points. Therefore, the interpolation process is independent of the SPM computation.

From that, an iterative method to compute multiple order reflections and/or diffractions has been developed. The method consists, basically, of calculating iteratively the induced current in each surface involved in the reflection to obtain the scattered field at the reflection point. In each surface the induced current is expressed in terms of a current
mode. The definition of a current mode is an exponential function whose amplitude and phase are smooth functions that can be easily interpolated from their values in a reduced number of sampling points. The current in the sampling points in a surface (passive surface) is obtained from the fields at that points due the current mode defined in a surface (active surface) that is illuminating by reflection the passive surface. These fields are computed by the SPM and as it will be shown. To perform this computation it is only necessary the minimization of several functions of two variables, avoiding the minimization of functions of more than two variables necessary in the inverse methods and the shouting of a large amount of rays necessary in the SBR, reducing consequently the computation time as will be illustrated in the Results section. Once the current mode in the passive surface has been obtained, this surface is considered as the active surface and therefore it will illuminate a new surface (the new passive surface) in the next step of an iterative procedure to solve the multiple iteration problems arising in scattering and radiation problems with complex bodies.

The proposed method is advantageous with respect to the SBR because it provides the possibility of the computation of multiple interactions between large objects sampling the surface of the objects with a low amount of points, amount which is independent of the frequency. On the other hand with respect to the inverse methods presented in [10-12], the advantage is that the functions to minimize depends only on two variables, independently of the number of surfaces involved in the multiple interaction. These advantages are possible due to the most important technical combination of this paper: the combination of the SPM with the interpolation of the induced currents over a body by using Bézier surfaces.

It is important to bring out that the presented approach can be considered iterative in the sense that the PO current integrated by the SPM in the surface for a multiple interaction is computed iteratively for a surface taking into account the current of the surface considered in the previous iteration using in each surface the classical PO approach. There are other Iterative Physical Optics (IPO) approaches in the literature, which basically try to solve the Magnetic Field Integral Equation (MFIE) using IPO [18, 19] as an alternative of a matrix solution of the MFIE. The idea of our approach is not to solve any integral equation, but to provide an alternative to the classical ray methods used to compute higher order contributions in high frequency.

This paper is arranged as follows. Part 2 summarizes the procedure to obtain by interpolation the phase and amplitude functions that define a current mode. Part 3 shows how to compute the PO
integral due to a current mode for an observation point in the near or the far field by using the SPM. The iterative method considered for the computation of the multiple interactions between the different surfaces of a complex body is described in Part 4. Results for problems with double reflection, triple reflection and higher order reflection in a rectangular cavity are presented in Part 5 to show the performances of the proposed approach. Finally, in Part 6, the conclusions and the main features of the approach are summarized.

## II. INTERPOLATION OF THE INDUCED CURRENT BY BÉZIER SURFACES

As mentioned above, the interpolation of the induced current was outlined in [15] and consists basically in using, as parameter interpolating function for each current mode, a Bézier surface to interpolate each component of the amplitude vector and another to interpolate the phase function. To interpolate the bidimensional scalar function $\Phi$ from a set of $(\mathrm{m}+1) \cdot(\mathrm{n}+1)$ values of the function, the control points of the Bézier surface are obtained by solving the following equation:

$$
\begin{equation*}
\mathrm{b}=\mathrm{U}^{-1} \Phi \mathrm{~V}^{-1} \tag{2}
\end{equation*}
$$

where:

$$
\begin{gather*}
\mathrm{b}=\left[\begin{array}{ccc}
\overrightarrow{\mathrm{b}}_{00} & \cdots & \overrightarrow{\mathrm{~b}}_{0 \mathrm{n}} \\
\vdots & \ddots & \vdots \\
\overrightarrow{\mathrm{~b}}_{\mathrm{m} 0} & \cdots & \overrightarrow{\mathrm{~b}}_{\mathrm{mn}}
\end{array}\right]_{(\mathrm{m}+1)(\mathrm{n}+1)}  \tag{3}\\
\Phi=\left[\begin{array}{ccc}
\vec{\varphi}_{00} & \cdots & \vec{\varphi}_{0 \mathrm{n}} \\
\vdots & \ddots & \vdots \\
\vec{\varphi}_{\mathrm{m} 0} & \cdots & \vec{\varphi}_{\mathrm{mn}}
\end{array}\right]_{(\mathrm{m}+1)(\mathrm{n}+1)}  \tag{4}\\
\mathrm{U}=\left[\begin{array}{llll}
\mathrm{B}_{0}\left(\mathrm{u}_{0}\right) & \cdots & \mathrm{B}_{\mathrm{m}}\left(\mathrm{u}_{0}\right) \\
\vdots & \ddots & \vdots & \\
\mathrm{B}_{0}\left(\mathrm{u}_{\mathrm{m}}\right) & \cdots & \mathrm{B}_{\mathrm{m}}\left(\mathrm{u}_{\mathrm{m}}\right)
\end{array}\right]_{(\mathrm{m}+1)(\mathrm{m}+1)}  \tag{5}\\
\mathrm{V}=\left[\begin{array}{lll}
\mathrm{B}_{0}\left(\mathrm{v}_{0}\right) & \cdots & \mathrm{B}_{\mathrm{n}}\left(\mathrm{v}_{0}\right) \\
\vdots & \ddots & \vdots \\
\mathrm{B}_{0}\left(\mathrm{v}_{\mathrm{n}}\right) & \cdots & \mathrm{B}_{\mathrm{n}}\left(\mathrm{v}_{\mathrm{n}}\right)
\end{array}\right]_{(\mathrm{n}+1)(\mathrm{n}+1)} \tag{6}
\end{gather*}
$$

$\overrightarrow{\mathrm{b}}_{\mathrm{ij}}$ being the control points which define the surface which interpolates the function $\Phi, \vec{\varphi}_{\mathrm{ij}}$ the sampling points of that function, and $B_{i}$ the Bernstein polynomials in terms of which the Bézier surface is expressed [13,14].

The interpolation can be performed taking $8 \times 8$ samples over the surface for a good representation of the current, taking into account the behavior of Bezier
surfaces as described in [13]. The samples are usually chosen equally spaced, although it is not a mandatory condition.

## III. CALCULATION OF A SIMPLE REFLECTION USING THE STATIONARY PHASE METHOD AND CURRENT MODES

The use of the SPM to obtain the radiated field of an antenna in presence of a convex object and calculate the PO integral was shown in [12]. SPM is a mathematical approach that is especially suitable to calculate integrals with rapid oscillation of the integrand phase. This situation is given for the PO integral in high frequency. For this reason, SPM is advantageous for high frequency electromagnetic analysis with respect to classical numerical techniques such as the Gauss quadratures. The SPM requires the search of a set of critical points: stationary phase or internal points, boundary points and vertex points which give the first, second and third order contribution to the PO integral. In this section, we will concentrate on the application of the current modes to obtain the first order contribution, i.e., the contribution made by the stationary phase points, to the PO integral. The other contributions could be obtained in a similar way.

Consider a body in which the electric and magnetic currents have been defined in terms of current modes. The scattered field at an observation point $\overrightarrow{\mathrm{r}}$ can be obtained by computing the PO integral on the surface $S^{\prime}$ of that object:

$$
\begin{align*}
\overrightarrow{\mathrm{E}}^{\mathrm{S}}(\overrightarrow{\mathrm{r}})= & \overrightarrow{\mathrm{E}}_{\mathrm{J}}^{\mathrm{S}}(\overrightarrow{\mathrm{r}})+\overrightarrow{\mathrm{E}}_{\mathrm{M}}^{\mathrm{S}}(\overrightarrow{\mathrm{r}})= \\
= & -j \lambda^{-1}\left(\int_{\mathrm{S}^{\prime}}\left(\hat{\mathrm{k}}_{\mathrm{s}} \times \overrightarrow{\mathrm{J}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right) \times \hat{\mathrm{k}}_{\mathrm{s}}\right) \frac{\mathrm{e}^{j \overrightarrow{\mathrm{k}}_{\mathrm{s}}\left(\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right)}}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|} \mathrm{dS}\right)+ \\
& +j \eta \lambda^{-1}\left(\int_{\mathrm{S}^{\prime}}\left(\hat{\mathrm{k}}_{\mathrm{s}} \times \overrightarrow{\mathrm{M}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)\right) \frac{\mathrm{e}^{\mathrm{j} \overrightarrow{\mathrm{k}}_{\mathrm{s}}\left(\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right)}}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|} \mathrm{dS}\right) \tag{7}
\end{align*}
$$

where $\vec{E}_{J}^{S}(\overrightarrow{\mathrm{r}})$ and $\overrightarrow{\mathrm{E}}_{\mathrm{M}}^{\mathrm{S}}(\overrightarrow{\mathrm{r}})$ are the contributions of the induced electric and magnetic current mode $\vec{J}\left(\vec{r}^{\prime}\right)$ and $\vec{M}\left(\vec{r}^{\prime}\right)$ to the radiated field, $\lambda$ is the wavelength, $\eta$ is the impedance in free space and $\hat{\mathrm{k}}_{\mathrm{s}}$ is the direction of observation for far field or the unit vector which joins the point $\overrightarrow{\mathrm{r}}^{\prime}$ over the surface with the observation point for near field $\left(\frac{\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|}\right)$. The induced currents can be expressed as follows:

$$
\begin{align*}
& \overrightarrow{\mathrm{J}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)=\hat{\mathrm{n}} \times\left[\begin{array}{cc}
1-\Gamma_{\mathrm{s}} & 0 \\
0 & 1-\Gamma_{\mathrm{h}}
\end{array}\right]\left[\begin{array}{l}
\mathrm{H}_{\mathrm{S}}^{\mathrm{i}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right) \\
\mathrm{H}_{\mathrm{h}}^{\mathrm{h}}\left(\mathrm{r}^{\prime}\right)
\end{array}\right]  \tag{8}\\
& \overrightarrow{\mathrm{M}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)=-\hat{\mathrm{n}} \times\left[\begin{array}{cc}
1+\Gamma_{\mathrm{s}} & 0 \\
0 & 1+\Gamma_{\mathrm{h}}
\end{array}\right]\left[\begin{array}{l}
\mathrm{E}_{\mathrm{s}}^{\mathrm{i}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right) \\
\mathrm{E}_{\mathrm{h}}^{\mathrm{i}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)
\end{array}\right] \tag{9}
\end{align*}
$$

where $\hat{\mathrm{n}}$ is the unit vector normal to the surface at point $\overrightarrow{\mathrm{r}}^{\prime}, \Gamma_{\mathrm{s}}$, and $\Gamma_{\mathrm{h}}$ are the Fresnel reflection coefficients [4], and $\mathrm{H}_{\mathrm{s}, \mathrm{h}}^{\mathrm{i}}$ and $\mathrm{E}_{\mathrm{s}, \mathrm{h}}^{\mathrm{i}}$ are the soft and hard components (perpendicular and parallel components respectively, see [4]) of the incident magnetic and electric fields at that point of the surface.

The two terms $\overrightarrow{\mathrm{E}}_{\mathrm{J}}^{\mathrm{S}}(\overrightarrow{\mathrm{r}})$ and $\overrightarrow{\mathrm{E}}_{\mathrm{M}}^{\mathrm{S}}(\overrightarrow{\mathrm{r}})$ of equation (7) can be written as follows, expressing the current by means of the amplitude and phase terms:

$$
\begin{equation*}
\overrightarrow{\mathrm{E}}_{\mathrm{J}}^{\mathrm{S}}(\overrightarrow{\mathrm{r}})=-\mathrm{j} \lambda^{-1}\left(\int_{S^{\prime}}\left(\hat{\mathrm{k}}_{\mathrm{s}} \times \overrightarrow{\mathrm{J}}_{0}\left(\overrightarrow{\mathrm{r}}^{\prime}\right) \times \hat{\mathrm{k}}_{\mathrm{s}}\right) \frac{\mathrm{e}^{\mathrm{jk}\left[\hat{\mathrm{k}}_{\mathrm{s}}\left(\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right)+\phi\left(\mathrm{r}^{\prime}\right)\right]}}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|} \mathrm{dS}\right) \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
\overrightarrow{\mathrm{E}}_{\mathrm{M}}^{\mathrm{S}}(\overrightarrow{\mathrm{r}})=\mathrm{j} \eta \lambda^{-1} \int_{\mathrm{S}^{\prime}}\left(\hat{\mathrm{k}}_{\mathrm{s}} \times \overrightarrow{\mathrm{M}}_{0}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)\right) \frac{\mathrm{e}^{\mathrm{jk}\left[\hat{\mathrm{k}}_{\mathrm{s}}\left(\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right)+\phi\left(\mathrm{r}^{\prime}\right)\right]}}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|} \mathrm{dS} \tag{11}
\end{equation*}
$$

where $\overrightarrow{\mathrm{J}}_{0}\left(\overrightarrow{\mathrm{r}}^{\prime}\right), \overrightarrow{\mathrm{M}}_{0}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)$ are the amplitude values of the electric and magnetic current respectively at a point $\overrightarrow{\mathrm{r}}^{\prime}$ on the surface, and $\phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)$ is the phase. These functions define the corresponding current mode and it is assumed they are approximated by the Bézier surface described in part 2.

To apply the SPM, the amplitude functions can be expressed as:

$$
\begin{align*}
& \mathrm{g}_{\mathrm{J}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)=\frac{\hat{\mathrm{k}}_{\mathrm{S}} \times \overrightarrow{\mathrm{J}}_{0}\left(\overrightarrow{\mathrm{r}}^{\prime}\right) \times \hat{\mathrm{k}}_{\mathrm{S}}}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|}  \tag{12}\\
& \mathrm{g}_{\mathrm{M}}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)=\frac{\hat{\mathrm{k}}_{\mathrm{S}} \times \overrightarrow{\mathrm{M}}_{0}\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|} \tag{13}
\end{align*}
$$

and the phase function is expressed as:

$$
\begin{align*}
\mathrm{f}\left(\overrightarrow{\mathrm{r}}^{\prime}\right) & =\hat{\mathrm{k}}_{\mathrm{s}}\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|+\phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)= \\
& =\left\{\begin{array}{cc}
\hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}^{\prime} \quad \text { for observation in far field } \\
\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right| & \text { for observation in near field. }
\end{array}\right. \tag{14}
\end{align*}
$$

Two integrals must be solved, one for the electric current and the other for the magnetic current. In parametric coordinates we have:

$$
\begin{align*}
& I_{J}=\int_{u=0}^{u=1} \int_{v=0}^{v=1} g_{J}(u, v) e^{j k f(u, v)} d u d v  \tag{15}\\
& I_{M}=\int_{u=0}^{u=1} \int_{v=0}^{v=1} g_{M}(u, v) e^{j k f(u, v)} d u d v \tag{16}
\end{align*}
$$

Both integrals satisfy the conditions for the application of the SPM method [16,17]. The first step in solving the integrals is to find the critical points. As mentioned above, only the internal points will be considered here. These are the points with the parametric coordinates $\left(u_{s}, v_{s}\right)$, where function $f(u, v)$ has a minimum, i.e., where the parametric derivatives at this point are zero:

$$
\left\{\begin{array}{l}
\mathrm{f}_{\mathrm{u}}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)=\frac{\partial \mathrm{f}}{\partial \mathrm{u}}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)=0  \tag{17}\\
\mathrm{f}_{\mathrm{v}}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)=\frac{\partial \mathrm{f}}{\partial \mathrm{v}}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)=0
\end{array}\right.
$$

The derivatives of the function $f(u, v)$ are:
$f_{u}(u, v)=\left\{\begin{array}{l}\frac{\partial \hat{\mathbf{k}}_{s} \cdot \vec{r}^{\prime}}{u}+\frac{\partial \phi\left(\vec{r}^{\prime}\right)}{u} \text { far field observation } \\ \frac{\partial\left|\vec{r}-\vec{r}^{\prime}\right|}{u}+\frac{\partial \phi\left(\vec{r}^{\prime}\right)}{u} \text { near field observation }\end{array}\right.$
$f_{v}(u, v)=\left\{\begin{array}{l}\frac{\partial \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}^{\prime}}{\mathrm{v}}+\frac{\partial \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{v}} \text { far field observation } \\ \frac{\partial\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\right|}{\mathrm{v}}+\frac{\partial \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{v}} \text { near field observation }\end{array}\right.$
where

$$
\begin{align*}
& \begin{aligned}
\frac{\partial\left|\vec{r}-\vec{r}^{\prime}\right|}{\partial u} & =\frac{-2\left(x-x^{\prime}\right)^{2} \frac{\partial x^{\prime}}{\partial u}-2\left(y-y^{\prime}\right)^{2} \frac{\partial y^{\prime}}{\partial u}-2\left(z-z^{\prime}\right)^{2} \frac{\partial z^{\prime}}{\partial u}}{2 \sqrt{\left(x-x^{\prime}\right)^{2}+\left(y-y^{\prime}\right)^{2}+\left(z-z^{\prime}\right)^{2}}} \\
& =-\hat{k}_{s} \cdot \frac{\partial \vec{r}^{\prime}}{\partial u} \\
\frac{\partial\left|\vec{r}-\vec{r}^{\prime}\right|}{\partial v} & =\frac{-2\left(x-x^{\prime}\right)^{2} \frac{\partial x^{\prime}}{\partial v}-2\left(y-y^{\prime}\right)^{2} \frac{\partial y^{\prime}}{\partial v}-2\left(z-z^{\prime}\right)^{2} \frac{\partial z^{\prime}}{\partial v}}{2 \sqrt{\left(x-x^{\prime}\right)^{2}+\left(y-y^{\prime}\right)^{2}+\left(z-z^{\prime}\right)^{2}}} \\
& =-\hat{k}_{s} \cdot \frac{\partial \vec{r}^{\prime}}{\partial v}
\end{aligned}
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}^{\prime}}{\partial \mathrm{u}}=\hat{\mathrm{k}}_{\mathrm{s}} \cdot \frac{\partial \overrightarrow{\mathrm{r}}^{\prime}}{\partial \mathrm{u}}  \tag{22}\\
& \frac{\partial \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}^{\prime}}{\partial \mathrm{v}}=\hat{\mathrm{k}}_{\mathrm{s}} \cdot \frac{\partial \overrightarrow{\mathrm{r}}^{\prime}}{\partial \mathrm{v}} \tag{23}
\end{align*}
$$

$\frac{\partial \phi\left(\vec{r}^{\prime}\right)}{u}$ and $\frac{\partial \phi\left(\vec{r}^{\prime}\right)}{\mathrm{v}}$ can be easily obtained as the derivatives of the current mode and $\frac{\partial \overrightarrow{\mathrm{r}}^{\prime}}{\mathrm{u}}$ and $\frac{\partial \overrightarrow{\mathrm{r}}^{\prime}}{\mathrm{v}}$ can be obtained as the derivatives of the Bézier surface which describes the scattered object. The expressions for the derivatives of a Bézier surfaces can be seen in reference [13].

Then, if the derivatives of the Bézier surface which describes the body are denoted as $\overrightarrow{\mathrm{r}}_{\mathrm{u}}{ }_{\mathrm{u}}$ and $\overrightarrow{\mathrm{r}}_{\mathrm{V}}{ }_{\mathrm{v}}$, the system of equations to solve is the following, corresponding the sign + to observation in far field and the sign - to observation in near field:

$$
\left\{\begin{array}{l}
\left(\frac{\partial \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{u}}\right) \pm\left.\hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{u}}^{\prime}\right|_{\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)}=0  \tag{24}\\
\left(\frac{\partial \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{v}}\right) \pm\left.\hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{v}}^{\prime}\right|_{\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)}=0 .
\end{array}\right.
$$

The Conjugate Gradient Method [20] has been used to solve the system of equations in our examples. Once the critical point has been obtained, its contribution to the PO integral is expressed as follows [12]:

$$
\begin{align*}
& \overrightarrow{\mathrm{I}}_{\mathrm{J}}=\frac{\pi}{\mathrm{k}} \frac{\mathrm{e}^{\mathrm{jk}\left[\overrightarrow{\mathrm{k}}_{\mathrm{s}}\left(\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)\right)+\phi\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)\right]}}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)\right|} \\
& \times \frac{\hat{\mathrm{k}}_{\mathrm{S}} \times \overrightarrow{\mathrm{J}}_{0}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right) \times \hat{\mathrm{k}}_{\mathrm{S}}}{\sqrt{\left|\mathrm{f}_{\mathrm{uu}}^{\mathrm{S}} \cdot \mathrm{f}_{\mathrm{vv}}^{\mathrm{S}}-\left(\mathrm{f}_{\mathrm{uv}}^{\mathrm{S}}\right)^{2}\right|}} \mathrm{e}^{\mathrm{j} \frac{\pi}{4} \sigma(\delta+1)}  \tag{25}\\
& \overrightarrow{\mathrm{I}}_{\mathrm{M}}=\frac{\pi}{\mathrm{k}} \frac{\mathrm{e}^{\mathrm{jk}\left[\overrightarrow{\mathrm{k}}_{\mathrm{s}}\left(\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)\right)+\phi\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)\right]}}{\left|\overrightarrow{\mathrm{r}}-\overrightarrow{\mathrm{r}}^{\prime}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)\right|} \\
& \times \frac{\hat{\mathrm{k}}_{\mathrm{S}} \times \overrightarrow{\mathrm{M}}_{0}\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)}{\sqrt{\left|\mathrm{f}_{\mathrm{uu}}^{\mathrm{S}} \cdot \mathrm{f}_{\mathrm{vv}}^{\mathrm{S}}-\left(\mathrm{f}_{\mathrm{uv}}^{\mathrm{S}}\right)^{2}\right|}} e^{\mathrm{j} \frac{\pi}{4} \sigma(\delta+1)} \tag{26}
\end{align*}
$$

with
$\sigma=\operatorname{sign}\left(f_{u v}^{S}\right)$ and $\delta=\operatorname{sign}\left[f_{u u}^{S} \cdot f_{v v}^{S}-\left(f_{u v}^{S}\right)^{2}\right]$,
$f_{u u}^{S}, f_{v v}^{S}$ and $f_{u v}^{S}$ being the second derivatives of the phase at the point $\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)$. These are easily computed, deriving (18) and (19):

$$
\begin{align*}
& \mathrm{f}_{\mathrm{uu}}^{\mathrm{S}}=\left.\frac{\partial}{\mathrm{u}}\left(\frac{\partial \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{u}} \pm \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{u}}^{\prime}\right)\right|_{\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)} \\
& \left.\begin{aligned}
& \frac{\partial^{2} \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{u}^{2}} \pm \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{uu}}^{\prime} \pm \frac{1}{\left|\overrightarrow{\mathrm{k}}_{\mathrm{s}}\right|} \times \mid\left\{\left(\overrightarrow{\mathrm{r}}_{\mathrm{u}}^{\prime} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{u}}^{\prime}\right)-\left(\overrightarrow{\mathrm{r}}_{\mathrm{u}}^{\prime} \cdot \hat{\mathrm{k}}_{\mathrm{s}}\right)^{2}\right\}
\end{aligned}\right|_{\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)}  \tag{27}\\
& \mathrm{f}_{\mathrm{vv}}^{\mathrm{S}}=\left.\frac{\partial}{\mathrm{v}}\left(\frac{\partial \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{v}} \pm \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{v}}^{\prime}\right)\right|_{\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)} \\
& =\frac{\partial^{2} \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{v}^{2}} \pm \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{vv}}^{\prime} \pm \frac{1}{\left|\overrightarrow{\mathrm{k}}_{\mathrm{s}}\right|} \times  \tag{28}\\
& \left.\left\{\left(\overrightarrow{\mathrm{r}}_{\mathrm{v}}^{\prime} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{v}}^{\prime}\right)-\left(\overrightarrow{\mathrm{r}}_{\mathrm{v}}^{\prime} \cdot \hat{\mathrm{k}}_{\mathrm{s}}\right)^{2}\right\}\right|_{\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)} \\
& \mathrm{f}_{\mathrm{uv}}^{\mathrm{S}}=\left.\frac{\partial}{\mathrm{u}}\left(\frac{\partial \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{\mathrm{v}} \pm \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{v}}^{\prime}\right)\right|_{\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)} \\
& \begin{aligned}
& \frac{\partial^{2} \phi\left(\overrightarrow{\mathrm{r}}^{\prime}\right)}{u v_{v}} \pm \hat{\mathrm{k}}_{\mathrm{s}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{uv}}^{\prime} \pm \frac{1}{\left|\overrightarrow{\mathrm{k}}_{\mathrm{s}}\right|} \times \\
= & \left.\left\{\left(\overrightarrow{\mathrm{r}}_{\mathrm{u}}^{\prime} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{v}}^{\prime}\right)-\left(\overrightarrow{\mathrm{r}}_{\mathrm{u}}^{\prime} \cdot \hat{\mathrm{k}}_{\mathrm{s}}\right)\left(\overrightarrow{\mathrm{r}}_{\mathrm{v}}^{\prime} \cdot \hat{\mathrm{k}}_{\mathrm{s}}\right)\right\}\right|_{\left(\mathrm{u}_{\mathrm{s}}, \mathrm{v}_{\mathrm{s}}\right)}
\end{aligned} \tag{29}
\end{align*}
$$

In equations (27-29) the sign + corresponds to observation in far field and the sign - to observation in near field. It is important to take into account the possibility of not all the surface is illuminated. In this case, only the illuminated part of the surface is considered to interpolate the current. Then, it appears a shadow boundary which contribution should be considered introducing a second order critical point as is shown in [12].

Equations (25) and (26) illustrate the advantage of the SPM with respect to other numerical integration techniques, because the value of the integral is reduced to the evaluation of a closed formula to evaluate the contribution of each critical point and the further summation of all the contributions. As it is stated in [12], there is, as much, only one first order critical point, four second order critical points and four third order critical points in each parametric surface that defines the geometry. On the other hand, there is necessary to evaluate the integrand for a high amount of points in a classical numerical integration technique (usually with an step of $\lambda / 10$ in the PO application, due to the rapid variation of the integrand's phase). Therefore, the number of operations involved in the calculation of the integral using the SPM is clearly inferior giving the computational advantage of this technique for integrals with rapidly variation integrand's phase,
typical situation in the analysis of electrically large electromagnetic problems.

It is also important to highlight that the position of the critical point is independent of the sampling points chosen to interpolate the current, being both procedures (the interpolation and the SPM computation) completely independent. In this section, the objective is only to show the applicability of the current interpolation to the analysis of scattered field by means by SPM, but there is not advantage with respect to the SPM computation without current interpolation. The advantage appears, as it is stated in next Section, when multiple interactions between different parts of a complex body must be considered to obtain the total scattered field.

An example is presented here to show the accuracy of the application of the current interpolation to the SPM. In the example, indicated in Figure 2, a comparison between the results obtained with the proposed approach and those obtained using a GTD/UTD is presented. The GTD/UTD results have been obtained with the code FASANT whose accuracy has been proven in [10]. The surface in Figure 2 is a quarter of a sphere whose sides are $2.35 \lambda$ $\mathrm{x} 2.35 \lambda$. The frequency is 300 MHz . The surface is a perfect electric conductor coated with an absorbing material that has both electric and magnetic losses with $\varepsilon_{\mathrm{r}}=2.5-\mathrm{j} 1.25, \mu_{\mathrm{r}}=1.6-\mathrm{j} 0.8$ and a thickness of $\tau=0.15 \lambda$. The electromagnetic illumination is by a vertical dipole placed at $12 \lambda$ from the center of the sphere. The dipole's coordinates are (2.0,3.0,0.0) and it is orientated according to the Z-axis of the reference system (X, Y, Z) depicted in Figure 2. The observation points are located along a line from $(4.0,2.0,0.0)$ to $(4.0,2.0,4.0)$. The minimum distance from the surface is $14.9 \lambda$. The comparison of the results of both methods is depicted in Figure 3, in which a close correspondence between both results can be observed.


Fig. 2. Spherical section illuminated by a dipole. Position of the observation points where the scattered field is computed.


Fig. 3. Scattered field by the spherical sector illuminated by a vertical dipole.

## IV. ITERATIVE METHOD FOR THE COMPUTATION OF HIGHER ORDER REFLECTIONS

When only single reflections are to be evaluated, the usage of the current interpolation approach, presented in Section 3 is not advantageous, because it replaces the minimization of the function distance that, in this case, depends on two variables, by the minimization of an interpolated phase function that depends also on two variables. Therefore, there is no gain in the minimization procedure. Moreover, the interpolation of the phase function requires the prior evaluation of the current at a set of sampling points and the corresponding interpolation using parametric interpolating surfaces. Therefore, the computation time is higher than when using a direct ray tracing to compute the stationary phase points, as stated in [12]. However, the application of a direct ray-tracing to obtain multiple reflections is much more complex, because it requires the minimization of a function with 2 n variables, which exponentially increases the computation with n , as was mentioned in the introduction.

The solution proposed in this paper consists of applying an iterative method in which the current induced over the surfaces involved in the multiple reflections is computed sequentially. Therefore, to obtain the current induced over a surface, it is necessary to know the current over the previous one. The interpolation method is used, as it only needs to store a small amount of information to accomplish this calculation.

Let us suppose a surface that we will call active surface, over which its induced current is defined by means of the current value at a set of control points. We will compute the induced current over another
surface (the passive surface), induced by the field radiated by the currents of the active surface. For that, a mesh of points over the passive surface is defined as depicted in Figure 4. In this mesh, we will compute the impressed field and from that the corresponding induced current. The mesh corresponds to the control points, which interpolate the current surface and the impressed field at each control point of the passive surface can be computed using the interpolated current of the active surface following the procedure described in Section 3.


Fig. 4. Interpolation of the current on the passive surface.

This procedure can be applied iteratively. Beginning at the source that illuminates a certain structure, the radiated field can be obtained in a given direction from the multiple reflections produced. The iterative method is carried out as follows:
a) The surfaces illuminated by the source are determined. These are the passive surfaces in the first iteration.
b) The impressed field at a set of sampling points is computed for each passive surface. From these values the equivalent currents are interpolated.
c) The next iteration starts. The passive surfaces become active.
d) The surfaces illuminated by the active surfaces are determined. These become the passive surfaces.
e) If it is the last iteration, it is checked. If not, the procedure is repeated from stage $b$.

An important task in the procedure is to select the passive surfaces for a given active surface. If there is not prior knowledge of which these surfaces are, all the surfaces of the model can be passive except the active. However, as it is stated in [21] the stationary phase points of the SPM correspond to the reflection points of Geometrical Optics. Therefore, the ray that joins the source with the stationary phase point and
the one that joins this point with the observation point must satisfy the Snell's law. From that, a previous selection of passive surfaces can be accomplished, determining the region of the space that satisfies the Snell's law from any point of the active surface. Only the surfaces places total or partially inside of this region can be passive surfaces. This previous selection allows reducing considerably the number of passive surfaces selected and, as a consequence, the time required for the iterative procedure. A similar reasoning can be done for the boundary and vertex critical points according of the properties of such points, described in [21].

An example with three surfaces illuminated by a plane wave with a direction of incidence $\hat{\mathrm{k}}_{\mathrm{i}}$, is depicted in Figure 5. Surface 1 is selected as the passive surface for the first step of the scheme, calculating the impressed field at each control point of the surface and from this the equivalent currents, which are denominated $\overrightarrow{\mathrm{J}}^{(1)}, \overrightarrow{\mathrm{M}}^{(1)}$. Once these currents have been determined, surface 1 becomes the active surface.


Fig. 5. First step of the iterative process. Computation of the induced currents by the incident plane wave.

The next step is to determine the radiation directions of the currents $\overrightarrow{\mathrm{J}}^{(1)}, \overrightarrow{\mathrm{M}}^{(1)}$ and to obtain the new passive surface. Surface 2 is the passive surface in our example. The impressed field over the control points of this surface is computed and the equivalent currents $\overrightarrow{\mathrm{J}}^{(2)}, \overrightarrow{\mathrm{M}}^{(2)}$ are obtained (Figure 6).


Fig. 6. Computation of the equivalent currents over surface 2 in the second step of the iterative process.

In the next step, surface 2 becomes the active surface and surface 3 the passive. The same procedure described above is applied and the equivalent currents on the surface $3\left(\overrightarrow{\mathrm{~J}}^{(3)}, \overrightarrow{\mathrm{M}}^{(3)}\right)$ are obtained. From these currents, the radiated field can be obtained (Figure 7).


Fig. 7. Scattered field in the direction $\hat{\mathrm{k}}_{\mathrm{s}}$ from the current calculated in the last step of the iterative process.

It is important to remark that to obtain the induced currents over the passive surfaces in each iteration, it is only necessary to calculate the radiated field by the previous surface (the active surface) from its induced currents. As these currents are represented by means of interpolated functions which depend only on the parametric coordinates of the active surface, each iteration requires several minimizations of functions of two variables, so many as sampling points are considered. Then, the minimization of a function of $n$ variables mentioned in the introduction is being replaced by an iterative procedure consisting in the minimization of functions of two variables. As the number of sampling points in each surface is not necessary to be big as was proved in [15] the number of this functions to minimize is small and the computation time is reduced drastically as will be shown in the Results section.

## V. RESULTS

In this section, the proposed method is validated comparing with the results obtained by the SPM without interpolation and by GTD/UTD for some simple cases. The comparison between the features both techniques (SPM and GTD/UTD) can be seen in [20]. These cases also illustrate the reduction in CPU time achieved with this method. After that, the application of the interpolation to a practical case is shown. The application considered is the computation of the RCS of a cavity. In this case, the proposed method is compared with the SBR, obtaining a considerable reduction in the CPU-time as it will be seen below.

The first case analyzed consists of the two surfaces indicated in Figure 8: one flat and other curved, with a curvature such that the normal vector at the surface turns at a maximum angle of $26^{\circ}$ when it moves along the surface. We will call this angle the "maximum turning angle" of the surface. Both surfaces in Figure 8 have sides of 4 m , which is equivalent to $13.33 \lambda$ and they are considered to be perfect electric conductors (PEC) coated with a material with electric and magnetic losses, characterized by a dielectric constant of $\varepsilon_{\mathrm{r}}=2.5-\mathrm{j} 1.25$, a magnetic constant of $\mu_{\mathrm{r}}=1.6-\mathrm{j} 0.8$ and a thickness of $0.15 \lambda$. The geometry is illuminated by a vertical dipole placed at point ( $0.0,-6.0,0.0$ ) as shown in Figure 8. The observation points were situated along a line from $(0.0,0.0,6.0)$ to $(0.0,4.0,4.0)$. Figure 9 shows a clear agreement between results obtained with the proposed method and those obtained with GTD/UTD.


Fig. 8. Planar surface with 4 meters per side and convex surface with 4 meters per side and maximum turning angle of $26^{\circ}$.


Fig. 9. Amplitude of the scattered field due to the double reflection between a planar surface and a convex surface coated with a material with losses.

The next geometry analyzed is depicted in Figure 10 and consists of three flat surfaces. The first is a PEC and the others are PECs coated with a material
with the same constants as the previous case. The plates have sides of 4 m (13.33 $\lambda$ ). A vertical dipole was placed at $(0.0,6.0,0.0)$ and the radiated field is obtained after the third reflection along a line from $(0.0,-5.0,0.0)$ to $(0.0,-9.0)$ consisting of 90 points. The comparison between the predicted values using our approach and those obtained with GTD/UTD is presented in Figure 11.


Fig. 10. Three planar surfaces.


Fig. 11. Amplitude of the scattered field due to the triple reflection produced by three planar surfaces, the first a PEC and the others PECs coated with a material with losses.

To demonstrate the efficiency of the proposed approach, we can compare the difference in CPU-time needed to analyze both a flat and a convex surface. Traditional SPM takes 33 seconds whilst the interpolation method takes just 5 seconds. Both results were obtained on a Pentium III with 1 GB of RAM. Taking into account that the CPU time for the field computation is practically the same, the difference is due to the advantage of the current interpolation. Traditional SPM has to minimize a function of four variables for each observation point ( 90 points in the example) in order to find the stationary phase points corresponding to each observation point. On the other hand, the interpolation method has to minimize a function of two variables to obtain the induced current in each sampling point of the second surface (64 points were used, 8 in each parametric direction) and another function of two variables to find the stationary phase point corresponding to each observation point. As the induced current only has to be computed once,

90 minimizations of a function of four variables are being replaced by 154 minimizations of functions of two variables. This fact allows for the time reduction mentioned above. Therefore, the method is efficient due to the reduction of the order of minimization.

Similar conclusions can be obtained with three flat surfaces where we replace 90 six-variables sets of minimizations by 218 two-variables sets of minimizations ( 64 for the induced current in the second surface, 64 for the induced current in the third surface and 90 to obtain the stationary phase point corresponding to each observation point). In this case, the GTD/UTD code FASANT used for the validation is unable to treat triple-reflections on curved surfaces, requires 15 minutes and 2 seconds to perform the analysis, while the interpolation method only needs 1 minute and 23 seconds.

Finally, as mentioned above, the result of the application of the proposed method to the analysis of the RCS of a cavity is shown. The case consists in a rectangular cavity whose dimensions are $30 \lambda$ x $10 \lambda$ x $10 \lambda$ (see Figure 12). The monostatic RCS was obtained varying the incidence angle for directions contained in a symmetry plane of the cavity, which contain the axis of the aperture. Figure 13 illustrates the comparison of the interpolation method with the SBR for the theta polarization. As can be seen there is a good agreement between both results. However, the SBR takes 12 hours, 12 minutes and 18 seconds, considering 10.000 ray tubes launched from the aperture (with a separation between points of a tenth of wavelength, that is to say, 100 points per square wavelength), while the interpolation method only requires 58 minutes and 47 seconds, because only 64 points are considered in the interpolation. It is important to notice that in this case the authors only try to prove that the computational cost is importantly reduced with the proposed method with respect to the SBR to obtain the same results. Logically, the diffraction should be included in both approaches to improve the results. The diffraction could be easily introduced in our approach considering the contribution of the second and third order critical points in the SPM formulation. The procedure would be identical to the one described in Section 2.


Fig. 12. Geometry of the rectangular cavity analyzed.


Fig. 13. Comparison between the results obtained by the interpolation method and the SBR for the monostatic RCS of the rectangular cavity.

## VI. CONCLUSIONS

A method for obtaining higher order contributions to the electromagnetic scattered field by complex bodies has been developed. The method is based on the interpolation of the induced currents, by means of Bézier surfaces. For each current mode, it uses one Bézier surface to interpolate each component of the amplitude and another one to interpolate the phase. Once it has interpolated these induced currents, the scattered field is obtained by solving the PO integral using the SPM.

The method has several advantages over others currently being used to obtain these higher order contributions. The method can be used with all surfaces not only flat ones like the Image Method. The number of sampling points required to interpolate the current on a surface is very low compared to the number of rays the SBR uses to solve these kinds of problems. Finally, the advantage the proposed approach over ray-tracing inverse methods is that it only needs to minimize functions of two variables to find the ray-path, irrespectively of the order of the contribution. Whilst inverse methods need minimize functions of 2 n variables, n being the order of the contribution, which means that the CPU-time needed for each minimization, increases exponentially with $n$.

The method developed is especially suitable for the analysis of problems where higher order contributions are of importance such as the propagation of tunnels or the computation of the RCS of cavities. An example of the last application has been shown in the Results section.

## ACKNOWLEDGEMENTS

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# The Use of Multiquadric Radial Basis Functions in Open Region Problems 

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#### Abstract

With the advent of neural networks, there has been a significant amount of research utilizing radial basis functions. In this paper, radial basis functions in conjunction with a meshless algorithm are used to solve electromagnetics problems in both open and closed regions. The algorithm for the solution of partial differential equations using the radial basis functions and development of the absorbing boundary condition will be discussed in detail. Several example problems will be discussed.


Keywords - radial basis functions, meshless algoirthms, absorbing boundary conditions, partial differential equation techniques

## I. INTRODUCTION

Radial basis functions (RBF) have received significant attention in the scientific literature over the past several years. Specifically, they have been investigated heavily in the field of neural networks. Until rather recently, RBF's have not been widely used in computational electromagnetics. They have been shown to have very good interpolation qualities and this has led to their recent use in inverse scattering methods in electromagnetics [1-11]. In this paper, radial basis functions in conjunction with a meshless algorithm are used to solve electromagnetics problems in both open and closed regions. In the next section, a discussion of the method used to solve partial
differential equations (PDE) using RBF's will be presented and an example will be shown. In Section III, the development of an absorbing boundary condition (ABC) based on the Wilcox expansion of the electric field will be presented and results utilizing the ABC will be shown. Finally, section IV will conclude the paper with discussions of some of the advantages and disadvantages of the proposed method.

## II. SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS USING RADIAL BASIS FUNCTIONS

There are several different RBF's that could be used to solve PDEs, but in this paper, the focus will be on the multiquadric functions. These functions have a function representation of

$$
\begin{equation*}
\sqrt{1+\frac{\left\|x-x_{j}\right\|^{2}}{c_{j}^{2}}} \tag{1}
\end{equation*}
$$

These functions have several interesting properties. The function's magnitude depends primarily upon the radial distance from its central location, $x_{j}$. This results in each slice of the function having a circular cross-section. In addition, these functions are continuous and have an infinite number of continuous derivatives at all points in space.
The algorithm employed in solving the PDEs in this research is termed a "meshless" method. This term is derived from the fact that
only information about nodes has to be known; in other words, no connectivity data between nodes is required. This is in contrast to the finite element method (FEM), which requires the knowledge of how the nodes are connected into elements. Generating a mesh for a detailed problem can be quite cumbersome and a misconnected node, which can easily go unnoticed, could lead to major problems. Obviously, since this is not necessary in this meshless algorithm, some of the problems associated with meshing can be alleviated. To discuss the details of the proposed algorithm, consider the closed region problem described by

$$
\begin{array}{ll}
\nabla^{2} E_{z}+k^{2} E_{z}=0 & \text { for }(x, y) \in \Omega  \tag{2}\\
E_{z}(x, y)=f(x, y) & \text { for }(x, y) \in \partial \Omega
\end{array}
$$

The problem domain along with the associated boundary conditions is illustrated in Fig. 1. The radial basis functions will be used as the expansion for the electric field such that

$$
\begin{gathered}
E_{z}=\sum_{j=1}^{N} u_{j} \sqrt{1+\frac{\left(x-x_{j}\right)^{2}+\left(y-y_{j}\right)^{2}}{c_{j}^{2}}} . \\
E_{z}(x, 1.0)=\sin \left(\frac{3 \pi x}{a}\right) \\
E_{z}(0.0, y)=0.0 E_{z}(2.0, y)=0.0 \\
E_{z}(x, 0.0)=0.0
\end{gathered}
$$

Fig. 1. Closed region problem domain.
Here, the $u_{j}$ 's are the unknown coefficients. The factor, $c_{j}$, is a scale factor for each RBF. In this research, all of the $c_{j}$ ' $s$ have been chosen to be equal to 0.25 . Now, the expansion (3) can be substituted into the original PDE (2) to obtain

$$
\begin{align*}
& \nabla^{2}\left[\sum_{j=1}^{N} u_{j} \sqrt{1+\frac{\left(x-x_{j}\right)^{2}+\left(y-y_{j}\right)^{2}}{c_{j}^{2}}}\right]+  \tag{4}\\
& k^{2} \sum_{j=1}^{N} u_{j} \sqrt{1+\frac{\left(x-x_{j}\right)^{2}+\left(y-y_{j}\right)^{2}}{c_{j}^{2}}}=0 .
\end{align*}
$$

At this point, there are N unknown coefficients (the $u_{j}$ 's) and only one equation. N equations are necessary to solve for the $\mathrm{N} u_{j}$ 's. The method used to obtain the N equations is called collocation. Collocation enforces either (4), or the boundary conditions, at the central points of the N RBF's. This yields a matrix equation

$$
\begin{equation*}
M u=b \tag{5}
\end{equation*}
$$

in which the matrix, $M$, will have elements

$$
M_{i, j}=\left[\begin{array}{l}
\nabla^{2} \sqrt{1+\left[\frac{\left(x_{i}-x_{j}\right)^{2}+\left(y_{i}-y_{j}\right)^{2}}{c_{j}}\right]}  \tag{6}\\
+k^{2} \sqrt{1+\left[\frac{\left(x_{-x}-x_{j}+\left(y_{i}-y_{j}\right)^{2}\right.}{c_{j}}\right]}
\end{array}\right] \text { for }\left(x_{i}, y_{i}\right) \text { in } \Omega(
$$

or

$$
\begin{equation*}
M_{i, j}=\sqrt{1+\left[\frac{\left(x_{i}-x_{j}\right)^{2}+\left(y_{i}-y_{j}\right)^{2}}{c_{j}}\right]} \text { for }\left(x_{i}, y_{i}\right) \text { on } \partial \Omega \text {. } \tag{7}
\end{equation*}
$$

The right-hand side vector will have elements

$$
b_{j}= \begin{cases}0 & \text { for }\left(\mathrm{x}_{\mathrm{i}}, y_{i}\right) \text { in } \Omega  \tag{8}\\ f\left(x_{i}, y_{i}\right) & \text { for }\left(\mathrm{x}_{\mathrm{i}}, y_{i}\right) \text { on } \partial \Omega\end{cases}
$$

and the column vector, $u$, will be the vector containing the coefficients for the RBF's.

To illustrate that accurate results can be obtained for the aforementioned problem, consider the case for which $\varepsilon=25.0$ and $\lambda=\frac{2 \pi}{k}=1.0$. Plots of both the numerical and analytic solution along the line $y=0.3$ (Fig. 2) and $x=0.5$ (Fig. 3) illustrate the accuracy that can be obtained using this method.


Fig. 2. Comparison of analytic and numerical solution (999 RBF's) along $\mathrm{y}=0$.

The solutions are obtained by employing 999 RBF's (33 equally spaced nodes along the xdirection and 33 equally spaced nodes along the $y$-direction) in the algorithm. In both cases, the numerical solutions lie nearly on top of the analytic solution.


Fig. 3. Comparison of analytic and numerical solution (999 RBF's) along $x=0.5$.

This is rather impressive considering that there are only 33 basis functions in the y-direction and there are 5 wavelengths of variation in the y-direction. An investigation of the RMS Error as the number of RBF's is varied is shown in Fig.4.


Fig. 4. RMS error as number of RBF's is varied.

This graph shows primarily what is expected. That is, as the number of RBF's is increased, the solution converges and the RMS error decreases. It should be pointed out that the computational cost of solving the matrix is higher than the traditional FEM for example, since the resulting matrix for this method is a full matrix. A direct solution technique (Gaussian elimination) was employed in the solution of the matrix equation. In addition to the disadvantage of having a full system matrix, the use of RBF's can result in illconditioning as the number of functions is increased. Ill-conditioning is primarily the problem for the increase in RMS error as the number of RBF's increases over 1000. A plot of the condition number as the number of RBF's is varied is shown in Fig. 5. It is important to realize that this graph is plotted on log axis; so, there are nearly 18 orders of magnitude difference between the smallest condition number and the largest condition number.

Further investigation of the condition number is required and some techniques to help alleviate the ill-conditioning will be presented in a separate paper. After seeing these results, though, it is clear that the RBF's


Fig. 5. Condition number as number of RBF's is varied.
can be used to solve closed region problems. In the next section, the focus will turn to that of the primary topic for this paper; that is, the discussion of the development of an ABC for an open region problem will be presented and then the ABC will be verified through several examples.

## III. DEVELOPMENT OF AN ABSORBING BOUNDARY CONDITION FOR USE WITH RADIAL BASIS FUNCTIONS

Consider the scattering of a transverse magnetic (TM) incident plane wave on a cylinder of radius one wavelength as shown in Fig. 6. The nodes (central points) for the RBF's will be placed in the shaded region and along the inner and outer boundaries. The outer boundary will be placed 1.35 wavelengths away from the center of the perfect electrically conducting (PEC) cylinder. The equation enforced at collocation points that are between the PEC cylinder and the outer boundary is the Helmholtz equation

$$
\begin{equation*}
\nabla^{2} E_{z}+k^{2} E_{z}=0 \tag{9}
\end{equation*}
$$



Fig. 6. Problem domain for circular PEC cylinder with circular outer boundary.

For collocation points that are on the circular cylinder, the equation enforced is

$$
\begin{equation*}
E_{z}=0 \text { for } r_{i}=\lambda \tag{10}
\end{equation*}
$$

and for nodes along the outer boundary, the equation enforced will be
$E_{z}(1.35, \phi)=A E_{Z}(1.30, \phi)+B E_{z}(1.25, \phi)$.
There are several aspects that should be pointed out with respect to the implementation of the ABC. Equation (11) states that the electric field at the outer boundary is proportional to the electric field at two points that lie along the same phi plane and are 0.05 and 0.1 wavelengths interior to the outer boundary. The fact that $E_{z}(1.25, \phi)$ and $E_{z}(1.30, \phi)$ need to be known does not mean that nodes have to be placed at these locations. Since the RBF's are entire domain basis functions, a representation of $E_{z}$ in terms of all of the RBF's can easily be found everywhere in the domain. Therefore, nodes are not necessary at $E_{z}(1.25, \phi)$ and $E_{z}(1.30, \phi)$. Obviously, the constants $A$ and $B$ in (11) must be obtained. It is first assumed that $E_{z}(1.25, \phi)$ and $E_{z}(1.30, \phi)$ can be approximated by the first two terms of the Wilcox expansion, which is

$$
\begin{equation*}
E_{z}(r, \phi)=\frac{e^{-j k r}}{\sqrt{r}}\left(C_{1}(\phi)+\frac{C_{2}(\phi)}{\rho}\right) . \tag{12}
\end{equation*}
$$

This assumption allows us to find $C_{1}$ and $C_{2}$ in terms of $E_{z}(1.25, \phi)$ and $E_{z}(1.30, \phi)$. Then, once these constants are known, the constants $A$ and $B$ can be found. After obtaining $A$ and $B$, an expression for $E_{z}$ along the outer boundary is known in terms of $E_{z}$ at points slightly interior to the boundary. Now, by using equations (9), (10), and (11), a matrix equation can be formed and the coefficients for the RBF's can be obtained. The algorithm will initially be tested using 2636 RBF's. Since the region between the cylinder and the outer boundary has an area equal to 2.5 square wavelengths, this problem domain has been discretized to yield approximately 32 functions/wavelength. In Figs. 7 and 8, the numerical and analytic solutions for the magnitude and the phase of the electric surface current on the PEC cylinder are compared. With this mesh density, a nodal average percent error was calculated to be $1.46 \%$.


Fig. 7. Magnitude of the current along the cylinder (~32 RBF/wavelength).

Since 32 functions/wavelength is a rather high mesh density, the results of a simulation in which the mesh density was lowered to approximately 23 functions/wavelength (1416 nodes) are presented in Figs. 9 and 10. Even at this density, the average percentage error was still very low at only $1.6 \%$. At this point,
the ABC has been verified to work properly for a circular outer boundary.


Fig. 8. Phase of the current along the cylinder ( $\sim 32 \mathrm{RBF} /$ wavelength).


Fig. 9. Magnitude of the current along the cylinder ( $\sim 23$ RBF/wavelength).

However, one of the problems often encountered with ABCs is that special meshing is required at the boundary so that the ABC can be implemented. For example, this could include placing the nodes close to the edge of the outer boundary along a constant phi plane or having a circular boundary, etc. To illustrate the robustness of the developed ABC and show that it does not have this requirement, the same cylinder will be
considered, but the outer boundary will be changed from a circular outer boundary to a square outer boundary. An illustration of the problem domain is shown in Fig. 11.


Fig. 10. Phase of the current along the cylinder (~23 RBF/wavelength).


Fig. 11. Problem domain for circular PEC cylinder with circular outer boundary.

The center of each side of the square boundary will be tangent to the circular boundary from the previous example. Thus, the square's sides will have lengths of 2.7 wavelengths and the region between the cylinder and outer boundary will have an area equal to 4.15 square wavelengths. A total of 2512 RBF's were used in the simulation. This resulted in a mesh density of approximately 25 functions / wavelength. Comparisons of the numerical and analytic solutions for the magnitude and phase are presented in Figs. 12 and 13, respectively. The average percent error per node was $2.08 \%$. From these graphs, one can conclude that switching from a
circular outer boundary to a square outer boundary did not significantly affect the results. This is particularly beneficial since it is desired to keep this method a "meshless" algorithm. We do not want to have to be very specific about where to put the RBF's in the problem domain and we also do not want to specify a specific type of outer boundary.


Fig. 12. Magnitude of the current along the cylinder ( $\sim 25 \mathrm{RBF} /$ wavelength) with square boundary.


Fig. 13. Phase of the current along the cylinder (~25 RBF/wavelength) with square boundary.

## IV. CONCLUSION

We have shown in this research a proof-of-
concept that a meshless method employing multiquadric radial basis functions and collocation can be used to obtain accurate results for both closed region and open region problems. Among the principle advantages of this method are the simplicity of the programming and the elimination of the need for sophisticated meshing. Among the disadvantages of this method is the fact that it yields a full matrix and that matrix can be illconditioned as the number of RBF's is increased. This problem is alleviated somewhat by the quickness of which each matrix element can be obtained. In addition, ill-conditioning can also result if care is not taken. Future research includes an investigation of the condition number and ways to prevent and alleviate the illconditioning problem that can result. Included in this investigation will be an investigation of node placement strategies as well as choice of the shape factor ( $\mathrm{c}_{\mathrm{val}}$ ); in addition, future work will also investigate what happens with discontinuities in the problem domain and how they should be handled.

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# A Novel Dual-band Small Size Microstrip Antenna 

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#### Abstract

A new small size dual-band microstrip antenna structure is proposed. The antenna consists of two resonant elements designed separately and integrated in such a manner to improve the compactness of the dual band structure and maintain the performance of each element. The structure is analyzed using lumped element approach and design curves are produced. The proposed approach is validated through a design example using theoretical curves and simulations. Implementation of an antenna operating in the 900 MHz and 1800 MHz bands is presented and comparisons between simulated and experimental results are given.


## I. INTRODUCTION

In many regions over the world the two operating frequency systems, GSM system operating at 900 MHz , and the DCS 1800 at 1800 MHz , are simultaneously used for mobile communications. Therefore, the design of dualband antennas for mobile handset applications has received much attention. Beside its dual-band operation, the antenna should also be small in size and excited by a single feed to meet the handset requirements. Due to their inherent flexibility, planar antennas are the most appropriate candidates to achieve these requirements. Recently, various types of planar inverted-F antennas (PIFAs) have been proposed for such applications [1]-[7]. Most of the previous works were based on simulator packages that use finite-difference time-domain (FDTD) or the method of moment. In this case the selection of the antenna dimensions for dual band operation becomes a tedious job.
In this paper, we propose a compact, dual band microstrip antenna that consists of two resonant elements. Each element can be designed separately at a specified center frequency and both elements integrated in such a way to maintain matching at the two frequencies, to have a compact structure and to provide negligible effect on each other. Each element consists of a short-circuited narrow line connected to a wider open circuited line as shown in Fig. 1. The lower frequency element (Fig. 1a) is a Tshaped patch and the high frequency element is an Lshaped patch (Fig. 1b). Both elements have a short circuited pin at the end of the narrow arm. The integrated dual band antenna is shown in Fig. 1c. The lumped
element model extracted from the transmission line theory is used to predict the resonant frequency of each element. The lumped element equivalence of the antenna elements is described in the next section where design curves are introduced. Discontinuities effects are explained in Section 3. In Section 4, we introduce specific design cases and conclusions are given in Section 5.

## II. LUMPED ELEMENT EQUIVALENCE OF THE PROPOSED ANTENNA ELEMENTS

The simplest microstrip resonator is a half wavelength line opened at both ends, or a quarter wavelength line opened at one end and short-circuited at the other. Unfortunately, the size of an antenna based even on a quarter wavelength line is physically too large to be used for handset GSM applications. For this reason, a shaped short circuit resonator such as short circuited H -structure [8] or Tstructure [9], which is considered as half of an open Hshaped antenna [10], has been developed as a compact antenna for single frequency operation. The H -shaped antenna has been analyzed using FDTD method, while the T-shaped one has been analyzed using the magnetic wall concept and transmission line theory.
The proposed dual-band antenna is shown in Fig. 1c and consists of two elements: a T-element that resonates at the lower frequency (Fig. 1a) and an L-element that resonates at the higher frequency (Fig. 1b). The transmission line equivalence of the short-circuited resonator elements in Figs. 1a and 1 b are shown in Figs. 2a and 2b, respectively. For the T-shaped element, $\theta_{1}$ and $\theta_{2}$ are the effective electrical lengths of the lines of physical lengths $l_{1}$ and $l_{2}$, respectively, after including the discontinuity effects. $Z_{1}$ and $Z_{2}$ are the characteristic impedances of the microstrip lines of widths $W_{1}$ and $W_{2}$, respectively. These transmission lines can be represented by the lumped elements L and C as shown in Fig. 2c. For the T-shaped element (Fig. 1a) the short circuited line is equivalent to a lumped inductance given by:

$$
\begin{equation*}
\omega \mathrm{L}=\mathrm{Z}_{1} \tan \theta_{1} \tag{1}
\end{equation*}
$$

where $\theta_{1}=\omega l_{1} / v_{\mathrm{ph}}, \mathrm{v}_{\mathrm{ph} 1}$ and $\mathrm{Z}_{1}$ are the phase velocity and characteristic impedance of a microstrip line of width $\mathrm{W}_{1}$ and effective length $l_{1}$. The capacitance of the two parallel
open lines is related to their physical dimensions by

$$
\begin{equation*}
\omega \mathrm{C}=2 \mathrm{Y}_{2} \tan \theta_{2} \tag{2}
\end{equation*}
$$

where $\theta_{2}=\omega l_{2} / \mathrm{v}_{\mathrm{ph} 2}, \mathrm{v}_{\mathrm{ph} 2}$ and $\mathrm{Y}_{2}$ are the phase velocity and characteristic admittance $\left(1 / \mathrm{Z}_{2}\right)$ of a microstrip line of width $\mathrm{W}_{2}$ and effective length $l_{2}$. Now combining (1) and (2) at the resonant frequency $\omega=\omega_{0}=1 / \sqrt{L C}$, we get

$$
\begin{equation*}
2 \tan \theta_{1} \tan \theta_{2}=\mathrm{K} \tag{3}
\end{equation*}
$$

where K is the ratio of the line impedances; $\mathrm{K}=\mathrm{Z}_{2} / \mathrm{Z}_{1}$. Using (3), $\theta_{2}$ is plotted against $\theta_{1}$ for different values of $K$ in Fig. 3a. It is observed that for a given value of $\theta_{1}, \theta_{2}$ decreases with decrease of K resulting in a reduction of the total antenna size.


Fig. 1. a) T-shaped antenna operates at the lower frequency, b) L-shaped antenna operates at the higher frequency, and c) Geometry of the proposed dual band antenna.

Similar analysis can be carried out for the L-shaped element. So let us denote by $\theta^{\prime}{ }_{1}$ and $\theta^{\prime}{ }_{2}$ the effective electrical lengths of the lines $l^{\prime}{ }_{1}$ and $l^{\prime}{ }_{2}$, respectively. $Z{ }_{1}$ and $Z^{\prime}{ }_{2}$ are the characteristic impedances of the microstrip lines of widths $W^{\prime}{ }_{1}$ and $W^{\prime}{ }_{2}$, respectively. Following the same steps as for the T-shaped element, at resonance we get:

$$
\begin{equation*}
\tan \theta_{1}^{\prime} \tan \theta_{2}^{\prime}=K . \tag{4}
\end{equation*}
$$

In this case K is equal to $\mathrm{Z}^{\prime}{ }_{2} / \mathrm{Z}^{\prime}{ }_{1}$. The design curves for this antenna are drawn in Fig. 3b for the same values of K as in Fig. 3a.


Fig. 2. a) Transmission line equivalence of antenna of Fig. 1a, b) Transmission line equivalence of antenna of Fig. 1b, and c) Lumped element equivalence of the resonators in Fig. 2(a) or 2(b).

(a)

(b)

Fig. 3. $\theta_{1}$ versus $\theta_{2}$ for different values of $K$. a) T-shaped antenna, and b) L-Shaped antenna.

In this case the total electrical length of the antenna is given by $\theta_{t}^{\prime}=\theta_{1}^{\prime}+\theta_{2}^{\prime}$. It is noted that for $K=1$ (uniform resonator), the total electrical length is $90^{\circ}$ while the total length decreases as K decreases.

Figs. 3a and 3b, which determine the electrical lengths of the antenna arms, are helpful for a primary design of the T and L shaped elements through judicial selection of the K factors. The antenna physical dimensions $l_{1}, l_{2}, \mathrm{~W}_{1}$, and $\mathrm{W}_{2}$ (primed and unprimed) can then be calculated for specific frequencies of operation as will be described in the next section.

## III. CIRCUIT LAYOUT

The circuit layout shown in Fig. 1, is the result of the conversion of the design parameters $Z_{1}, \theta_{1}, Z_{2}, \theta_{2}, Z_{1}^{\prime}, \theta_{1}^{\prime}$, $\mathrm{Z}^{\prime}$, and $\theta_{2}^{\prime}$, to a physical dimensions $\mathrm{W}_{1}, l_{1}, \mathrm{~W}_{2}, l_{2}, \mathrm{~W}^{\prime}, l_{1}^{\prime}$, $\mathrm{W}^{\prime}{ }_{2}$, and $l_{2}^{\prime}$ in a selected substrate material at a design frequency. However, for wide bandwidth requirements, microstrip antennas are usually implemented on a thick air substrate. In this case, the required discontinuities in the structure significantly affect the resonance frequency of the antenna. Microstrip discontinuity can be represented as an equivalent circuit at some point in the transmission line. The component values, of the equivalent circuit, depend on the parameter of the line and the discontinuity as well as the frequency of operation. In some cases the equivalent circuit involves a shift in the phase reference planes on the transmission lines [11]. One approach for eliminating the discontinuity effect is to construct an equivalent circuit, including it in the design of the circuit, and compensating for its effect by adjusting other circuit parameters. In our case, the effects will be compensated in the lengths of the lines using either closed form expressions or through the aid of the IE3D simulator. The main discontinuities that exist in the proposed structure are the open end, T junction, shorting post, and the bend.

## Open end

An empirical expression for the open-end effect is given in [12, Eq. 1]. It can be used to calculate the equivalent additional line length $\Delta l_{o c}$.

## Tee junction

The more useful representation of the Tee junction, for the design procedure, is to define the shift in reference planes as shown in Fig. 1a. The reference planes parameters $d_{1}$, and $d_{2}$ can be calculated in terms of the substrate parameters and the lines widths $\mathrm{W}_{1}$, and $\mathrm{W}_{2}$ from semiempirical expressions given in [13, p. 245].

## Shorting post

The shorting post is usually characterized by its equivalent inductance. This definition is not suitable to be incorporated in microtsrip antenna design since the
resonance frequency is highly dependent of the location of the post. For this reason the shorting post is studied and compensated using the IE3D simulator. This can be achieved, first, by calculating the phase angle of $\mathrm{S}_{11}$ of a short circuit line of characteristic impedance $\mathrm{Z}_{1}$ and electrical length $\theta_{1}$ from

$$
\begin{equation*}
\left.\operatorname{Ang}\left(S_{11}\right)\right|_{s c}=180-2 \tan ^{-1}\left(\frac{Z_{1} \tan \theta_{1}}{50}\right) \tag{5}
\end{equation*}
$$

After that, an equivalent line of width W and length $l$ terminated by a shorting post can easily be calculated with the optimization procedure tool of the IE3D. The optimization objective in this case is to match the theoretical angle from Eq. (5) to the angle of $S_{11}$ of the physical length $l$ at the design frequency.

## Bend

The bend is usually modeled in terms of its equivalent circuit [11], which is not easy to compensate for in the lines lengths. However, it is easy to compensate for the bend and the open end, simultaneously, of the L-shape through the optimization tool of the IE3D. This can easily be performed by equating the phase angle of $S_{11}$ of the open circuit layout in Fig. 4 c to the theoretical angle of the open circuited line given by

$$
\begin{equation*}
\left.\operatorname{Ang}\left(S_{11}\right)\right|_{o c}=-180+2 \tan ^{-1}\left(\frac{Z_{2} \cot \theta_{2}^{\prime}}{50}\right) \tag{6}
\end{equation*}
$$

For thin conventional dielectric materials, where, $2<\varepsilon_{\mathrm{r}}<$ 10 and thickness $0.5<\mathrm{h}<2 \mathrm{~mm}$, the use of closed form expressions [11-13] for most of the microstrip discontinuities below 10 GHz leads to very good results.

## IV. DESIGN CASE

The design approach described in the previous section is now validated through the design of an antenna that operates in the $900 / 1800 \mathrm{MHz}$ bands. An air substrate of thickness 6.4 mm is used. Considering the T-shaped element, the thin line width $\mathrm{W}_{1}$ is selected to be 2 mm . This small value helps to reduce any parasitic loading between the T and L elements after their integration. The resultant characteristic impedance of this line is $Z_{1}=193$ $\Omega$. Now, if we take the impedance ratio $K=0.6$, the characteristic impedance of the wide line, $\mathrm{Z}_{2}$ will be 115.8 $\Omega$, corresponding to about 7.7 mm line width $\mathrm{W}_{2}$. Fig. 3a is now used to compromise between $\theta_{1}$ and $\theta_{2}$. Taking $\theta_{1}$ $=25^{\circ}, \theta_{2}$ is $32.8^{\circ}$. A probe feed of radius 0.5 mm is located close to the shorting post of radius 0.3 mm . For ideal lines with no discontinuity effects, the physical lengths of this element calculated at 900 MHz correspond to $25^{\circ}$ and $32.8^{\circ}$ would be 23.2 mm and 30.4 mm ,
respectively. The physical dimensions shown in Fig. 1a are calculated as follows:
$\Delta l_{o c}=3.1 \mathrm{~mm}[12$, Eq. 1].
$\mathrm{d}_{1}$ and $\mathrm{d}_{2}$ are computed as 0.4 mm and 3.2 mm , respectively by use of [13, p. 245], while $l_{2}=30.4-3.1=$ 27.3 mm .
$l_{1}$ is obtained from the IE3D to match the theoretical phase angle of $\mathrm{S}_{11}$, calculated from Eq. (5) as $58.1^{\circ}$, to the angle of the line terminated by a shorting post of radius 0.3 mm located as shown in Fig. 4a. The optimum length for $l_{1}$ is 19.4 mm . This means that, the effect of the shorting post at this position is equivalent to a line of the same width 2 mm and 3.8 mm length at 900 MHz .

The simulated resonance frequency corresponding to these dimensions is 935 MHz . This little variation is due to the neglection of feeder effect and the accuracy of the model used to calculate the reference planes. Slight increase of $l_{1}$ to 19.7 mm and $l_{2}$ to 28.6 mm reduces the resonance frequency to 908 MHz . In order to do a further reduction of the area occupied, the right arm of the T element is bent parallel to the narrow arm as shown in the inset of Fig. 5a. The effect of the bend is accounted for by using the simulator, and the simulation results of $\left|\mathrm{S}_{11}\right|$ are given in Fig. 5a. The resonant frequency of the T- shaped element is 908 MHz and the bandwidth, corresponding to a 10 dB return loss, is 8 MHz .


Fig. 4. Circuit layout elements to calculate discontinuities effect (a) the shorting post for the T-shaped at 900 MHz , (b) the shorting post of the L-shaped at 1800 MHz , and (c) the bend and open circuit effect of the L-shaped at 1800 MHz .

Similar steps are used to design the L-shaped antenna element at 1800 MHz . The narrower line width is selected to be $\mathrm{W}^{\prime}{ }_{1}=3 \mathrm{~mm}$ which corresponds to a characteristic impedance of $169 \Omega$. Taking the impedance ratio $\mathrm{K}=0.5$, the characteristic impedance of the wider line is $84.5 \Omega$ which requires a line width of 14 mm . Taking $\theta_{1}{ }^{\prime}=28^{\circ}$ in Fig. $3 \mathrm{~b}, \theta_{2}{ }^{\prime}$ is $43^{\circ}$. For ideal transmission line elements,
the physical lengths would be 13 mm , and 19.9 mm for narrow and wide lines, respectively. The physical dimensions of the L-shaped circuit layout shown in Fig. 1b are calculated as follow:
The length of the narrow line ended by a shorting post of 0.3 mm radius, shown in Fig. 4b, is calculated by equating the phase angle of $\mathrm{S}_{11}$ from the IE3D with the theoretical value from Eq. (5) for $Z^{\prime}{ }_{1}$ and $\theta^{\prime}{ }_{1}$ at 1800 MHz . The resultant $l^{\prime}$ is 6.5 mm . This means that the shorting post at this position is equivalent to a line of 3 mm width and 6.5 mm length.


Fig. 5. Simulated $S_{11}$ of the separate $T$ and $L$ shaped elements designed on air substrate of 6.4 mm thickness, a) Simulation results of the T-shape element designed at 900 MHz , and b) Simulation results of the L-shape element designed at 1800 MHz .

The bend is then calculated by the help of the optimization procedure of the IE3D as described in the previous section. The theoretical angle of the open circuited line from Eq. (6) is $-57.8^{\circ}$. The circuit layout proposed is shown in Fig. 4 c . The short length $\Delta l$ is chosen to be 1.8 mm . The optimum value of the length $l^{\prime}{ }_{2}$ to match this angle is 11.5 mm . The simulated resonance frequency obtained based on these dimensions is 1870 MHz . The shift in the resonance frequency is due to the feeder effect. This can be simply modified by little increase in any length of the circuit. For $l_{2}=15.4 \mathrm{~mm}$ the simulated resonance frequency becomes 1787 MHz . The simulation results for the L-shaped element are shown in Fig. 5b where the 10 dB return loss bandwidth is 40 MHz at the center frequency.

Integration of the dual band antenna: Now the two elements are integrated as in Fig. 1c, while maintaining the same shorting post and feeder locations. The simulation results of the integrated structure are shown in Fig. 6. The lower band and upper band resonance occur now at $f_{L}=$ 907 MHz and $\mathrm{f}_{\mathrm{H}}=1835 \mathrm{MHz}$ with bandwidths of 8 MHz and 24 MHz respectively. It is thus seen that while the integration has almost no effect on the center frequency and bandwidth at the lower band, it caused an increase of the resonant frequency at the higher band by about 48 MHz , and a reduction in the bandwidth from 40 MHz to 24 MHz . We attribute this frequency shift and bandwidth reduction to the parasitic loading of the T- element on the L-element at the higher band. The simulated radiation patterns at the two resonant frequencies are plotted in Figs. 7 and 8. Both show an approximate omnidirectional radiation in the azimuth plane ( $\mathrm{x}-\mathrm{z}$ plane).


Frequency (GHz)

Fig. 6. Simulated $S_{11}$ of the integrated $T$ and $L$ shaped elements in Figs. 5a and 5b.

The antenna gain is computed to be about -0.15 dBi and 1.5 dBi for the lower and upper resonant frequencies, respectively. Such relatively low gains are typical for compact antennas [6], [14]. We note that the antenna efficiency at the lower band is about $70 \%$, so while the gain is -0.15 dBi the antenna directivity is equal to 1.8 dBi .


Fig. 7. Simulated radiation patterns at 907 MHz for the antenna in Fig. 5a.


Fig. 8. Simulated radiation patterns at 1835 MHz for the antenna in Fig. 5a.

Experimental work: The dual band antenna is built on a foam substrate with dielectric constant close to that of air. The ground plane used is a copper plate of dimensions 64 $\times 47 \mathrm{~mm}^{2}$. The experimental results are shown in Fig. 9 . The measured bandwidths based on 10 dB return loss, are about 12 and 31 MHz (compared to 8 MHz and 24 MHz from simulations) centered at 870 MHz and 1756 MHz , respectively. The observed change of the center frequencies and bandwidths from the simulation results
may be attributed to the inaccurate tools used for installing the feeder and short circuit. Simulations have shown that the resonance frequencies and bandwidths are sensitive to the locations of the short circuit and the feed. In addition, extra losses in the experiment, due to dielectric loss and finite ground plane, may be responsible for the increased experimental bandwidths compared to the simulation results.

Finally, to appreciate the size reduction achieved, we note that the built antenna occupies a volume of $27 \times 44 \times 6.4$ $\mathrm{mm}^{3}$ and mounted on a ground plane of $64 \times 47 \mathrm{~mm}^{2}$. In terms of the wavelength at $1800 \mathrm{MHz}(\lambda=166 \mathrm{~mm})$, the antenna volume is about $0.16 \lambda \times 0.26 \lambda \times 0.04 \lambda$ which makes it quite compact.


Fig. 9. Measured $\mathrm{S}_{11}$ of the integrated T and L shaped elements in Figs. 5a and 5b.

## V. CONCLUSION

A new small size dual-band antenna structure has been proposed. Namely a T-patch antenna element tuned at 900 MHz is integrated with an L-patch tuned at 1800 MHz . A simple design procedure based on lumped element representation is used to obtain design equations and curves that are used for a preliminary design. The design is trimmed by using the IE3D simulation tool to account for the end and bend effects. Simulation results show the feasibility of achieving a compact dual band antenna occupying a volume of $0.16 \lambda \times 0.26 \lambda \times 0.04 \lambda$, where $\lambda$ is the wavelength at the upper band $(1800 \mathrm{MHz})$. The simulation results are validated by building and testing an antenna prototype. Experimental and simulation results show reasonable agreement in terms of the center frequencies and bandwidths.

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# An Efficient Broadband Analysis of an Antenna via 3D FEM and Pade Approximation in the Frequency Domain 

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#### Abstract

The paper presents a finite element model for the calculation of the impedance of an antenna over a wide frequency band. The antenna has been designed to analyze a rectenna (rectifying antenna) in the context of wireless microwave energy transfer. The modelling approach combines a 3D edge element method (FEM) with a Padé approximation procedure. It allows to obtain an explicit expression of the impedance over a large frequency band. The comparison of the proposed technique with a standard finite element method shows that the computational cost is significantly reduced.


## I. INTRODUCTION

Microwave rectennas (rectifying antennas) are devoted to power transmission and detection. Applications include long distance power beaming, signal detection and wireless control systems. Power transmitting and receiving systems must be designed so that the power transmitted from the transmitting antenna is transmitted efficiently to the rectenna and is converted into DC power by rectifiers. Efficient field-circuit simulations are required in the design and characterization of such rectennas since non linear lumped elements are included. The knowledge of the input impedance of the antenna is of particular importance in evaluation the conversion efficiency.

In [1] a rectenna structure involving a loop antenna was studied. This rectenna is devoted to low-power applications. The targeted applications include microwave power reception from various sources in a large frequency range. The operating frequency may belong to the industrial, scientific and medical band (central frequency of 2.45 GHz ). Also investigations include radio-frequency identification (RFID) applications where wideband signals may be used. For these reasons a reliable circuit model of the rectenna allowing a global simulation over the band [0, 20] GHz was required. The circuit model takes into account both distributed electromagnetic portions of the antenna and the rectifier circuit. From the 3D electromagnetic modelling of the structure the input impedance was obtained as a function of the frequency. This technique provides an adequate way to incorporate the impedance into a non-linear circuit simulation. With such an approach the
impedance has to be calculated over a wide frequency band with a 3D modeling tool. This can be achieved as in [1] with a standard frequency domain method (boundary element method or finite element method for example).

A finite element model provides an efficient way for solving electromagnetic problems. In a frequency domain analysis the electromagnetic fields are discretized over a meshed volume. The unknowns are the solution of a linear system whose matrix depends on frequency. In a wide frequency band analysis the linear system has to be solved for each frequency of interest. This often leads to a huge computational cost. An alternative approach is to search for a power series expansion of the solution about a center frequency. The approach requires only one single matrix inversion. The radius of convergence is limited but it is possible to extend the interval using a corresponding Padé approximant. This technique is known as an asymptotic wave form analysis (AWE).

The AWE approach has been combined with integral equations in 3D [2]-[4] to solve scattering problems involving perfectly metallic obstacles. For solving general electromagnetic problems including inhomogeneous media and complex geometries, the finite element method (FEM) provides a powerful tool. The AWE approach used in connection with FEM was shown to deal with electromagnetic problems within bounded domains in [5]-[7] where passive microwave devices such as waveguides and cavities were studied. In these almost closed structures the boundaries of the studied domain consist of perfectly conducting walls or access planes. Then the efficiency of AWE relies on the fact that dominant poles and zeros of the network transfer function can be used to build rational approximations of the solution. Indeed in such a case the resonant modes, can be computed in a first step from a generalized eigenvalue problem and can be used in a second step to give an expansion of the solution. On the other hand, for electromagnetic problems in unbounded domains like radiation of an antenna in free space or scattering problems the fields cannot be expressed with resonant modes of the structure and efficient

Padé approximation are difficult to obtain. An extension of AWE combined with finite elements for radiation problems has been proposed in [8] for the 2D case. In a 2D analysis the structure is infinite along one direction and the electromagnetic problem reduces to a scalar wave problem (TM case or TE case). In such a configuration one of the two fields (electric or magnetic) has the same direction that the infinitely long structure. The ability of this method was demonstrated in the scattering of canonical obstacles having simple shapes.

In this work the AWE technique used in conjunction with finite elements is successfully extended in three dimensions (3D) for solving radiation problems in free space. The approach combines the vectorial finite element method and a Padé approximation. The numerical method is shown to provide a fast computation of the impedance of an antenna over a wide frequency band. The antenna is the loop antenna of the rectenna considered in [1] for which only a three dimensional analysis allows to obtain the distribution of the electromagnetic fields. The method is based on first order edge finite elements. A Silver-Müller boundary condition is used for the truncation of the domain. Once the finite element matrix has been built for one frequency an explicit expression (power series) of the fields and the impedance are available over a frequency band. From the power series a Padé approximation (rational function) can be derived. It is shown that a very good approximation is obtained even if several sharp resonance peaks are included in the studied range. The comparisons between the presented technique and a standard finite element analysis clearly underline the advantages of the proposed model.

## II. Electromagnetic problem

We consider the 3D problem of an antenna radiating in its surrounding medium. The dimensions of the studied device are shown in Fig. 1. The loop is assumed to be infinitely thin and perfectly conducting. However for the sake of generality the induced current density J is included in the equations since the presented analysis remains also valid in this case. The loop is excited by an impressed current $\mathrm{J}_{\mathrm{imp}}$ between the ends of the two arms.

For a full wave analysis we deal with the Maxwell equations in the frequency domain:

$$
\begin{align*}
\nabla \times \mathbf{E} & =-j \omega \mathbf{B}  \tag{1}\\
\nabla \times \mathbf{H} & =\mathbf{J}_{i m p}+\mathbf{J}+j \omega \mathbf{D}, \tag{2}
\end{align*}
$$

where E and H are the electric and magnetic field respectively, $\mu$ is the permeability, $\varepsilon$ is the permittivity.

The constitutive relations are given by:

$$
\begin{align*}
\mathbf{B} & =\mu \mathbf{H}  \tag{3}\\
\mathbf{D} & =\varepsilon \mathbf{E} . \tag{4}
\end{align*}
$$

The Ohm law gives:

$$
\begin{equation*}
\mathbf{J}=\sigma \mathbf{E} \tag{5}
\end{equation*}
$$

From equations (1-4), we establish the vector wave equation in terms of the electric field E :

$$
\nabla \times \frac{1}{\mu} \nabla \times \mathbf{E}+j \omega(\sigma+j \varepsilon \omega) \mathbf{E}=-j \omega \mathbf{J}_{i m p}
$$



Fig. 1. The loop antenna and its dimensions.

## III. Finite element formulation

A computation using the finite element method is performed in a finite region which includes the antenna and some of its surrounding medium. In order to truncate the volume of the computational domain the Silver-Müller condition is applied as an absorbing boundary condition [9]. It is given by:

$$
\begin{equation*}
n \times \nabla \times \mathbf{E}=j k \mathbf{E}_{\mathrm{tan}} \tag{7}
\end{equation*}
$$

where k is the wave number in free space and $\mathrm{E}_{\text {tan }}$ is the tangential electric field on the outer boundary surface.

This ABC preserves the sparsity and symmetric features of the final matrix. It is exact for normal incidence. Let denote $\Omega$ the computational domain and $\Gamma$ the outer boundary. As usual with FEM, we define the space of the work:

$$
\begin{equation*}
V=\left\{u \in\left(L^{2}(\Omega)\right)^{3}, \nabla \times u \in\left(L^{2}(\Omega)\right)^{3}\right\} . \tag{8}
\end{equation*}
$$

A weak formulation of the problem is obtained after multiplying the vector wave equation by a test function F in V :

$$
\begin{gather*}
\langle j \omega(\sigma+j \varepsilon \omega) \mathbf{E}, \mathbf{F}\rangle_{\Omega}+\left\langle\frac{1}{\mu} \nabla \times \mathbf{E}, \nabla \times \mathbf{F}\right\rangle_{\Omega}+ \\
\left\langle j \omega \sqrt{\frac{\varepsilon}{\mu}} \mathbf{E}, \mathbf{F}\right\rangle_{\Gamma}=-j \omega\left\langle\mathbf{J}_{i m p}, \mathbf{F}\right\rangle_{\Omega} \tag{9}
\end{gather*}
$$

where $\langle,\rangle_{\Omega}$ denotes the scalar product in $V$.
To solve equation (9) numerically, the domain is discretized with tetrahedral elements. The electric field can be written in terms of basis functions associated with the edges of these elements [10, 11]. From equation (9) and by using test functions F the same as interpolation functions (Galerkin method) we get:

$$
\begin{equation*}
\left(A_{0}+\omega A_{1}+\omega^{2} A_{2}\right) v=b \tag{10}
\end{equation*}
$$

where $v$ is the unknowns' vector, $b$ is the excitation currents vector and $A_{0}, A_{1}$, and $A_{2}$ are matrices which only depend on the mesh and on the medium.

## IV. AN EFFICIENT COMPUTATIONAL SCHEME FOR BROADBAND ANALYSIS

Consider an arbitrary $\omega_{0}$ such that $A_{0}$ is non-singular, the Taylor series expansion of the matrix polynomial in equation (10), about the frequency $\omega_{0}$ can be written as:

$$
\begin{equation*}
A(\omega)=\bar{A}_{0}+\left(\omega-\omega_{0}\right) \bar{A}_{1}+\left(\omega-\omega_{0}\right)^{2} \bar{A}_{2} \tag{11}
\end{equation*}
$$

where the matrices $A_{i},(i=0,1,2)$ can be obtained from equation (10) and equation (11):

$$
\begin{align*}
& \bar{A}_{0}=A_{0}+\omega_{0} A_{1}+\omega_{0}^{2} A_{2}  \tag{12a}\\
& \bar{A}_{1}=A_{1}+2 \omega_{0} A_{2} \\
& \bar{A}_{2}=A_{2} \tag{12c}
\end{align*}
$$

The solution vector $v(\omega)$ has power series representation about $\omega_{0}$, given by:

$$
\begin{equation*}
v(\omega)=\sum_{i=0}^{\infty} v_{i}\left(\omega-\omega_{0}\right)^{i} \tag{13}
\end{equation*}
$$

The power series representation of the excitation vector $b(\omega)$ is written as:

$$
\begin{equation*}
b(\omega)=\sum_{i=0}^{\infty} b_{i}\left(\omega-\omega_{0}\right)^{i} \tag{14}
\end{equation*}
$$

We can evaluate the coefficients of the power series of $\mathrm{v}(\omega)$ by the following procedure:

$$
\begin{align*}
& \left(\bar{A}_{0}+\left(\omega-\omega_{0}\right) \bar{A}_{1}+\left(\omega-\omega_{0}\right)^{2} \bar{A}_{2}\right)\left(v_{0}+\ldots\right. \\
& \left.v_{1}\left(\omega-\omega_{0}\right)+\ldots\right)=b_{0}+b_{1}\left(\omega-\omega_{0}\right)+\ldots \tag{15}
\end{align*}
$$

If we equate both sides of equation (15) term by term, we obtain the following iterative expression:

$$
\begin{equation*}
v_{i}=\bar{A}_{0}^{-1} b_{i}-\sum_{j=1, j \leq i}^{2} \bar{A}_{0}^{-1} \bar{A}_{j} v_{i-j} \quad, \quad i=0,1, \ldots \tag{16}
\end{equation*}
$$

It is very important to note that only a single inverse $\bar{A}_{0}^{-1}$ is needed in the iteration procedure.

## V. Padé approximation

A Padé approximation is derived by expanding a function as a ratio of two power series and determining both the numerator and denominator coefficients. Padé approximations are usually superior to Taylor expansion when functions contain poles, because the use of rational functions allows a good representation, and it provides an extension beyond the interval of convergence of the series [12]. The solution is expressed as a power series of the form:

$$
\begin{equation*}
v(\omega)=\sum_{i=0}^{\infty} v_{i}\left(\omega-\omega_{0}\right)^{i} \tag{17}
\end{equation*}
$$

where the coefficients $v_{i}, i=0,1,2, \ldots$. can be computed iteratively using (16). The expansion is convergent within the region $\left|\omega-\omega_{0}\right|<R$, where $R$ is the radius of convergence of this power series.

Since $v(\omega)$ is complex, and since the Padé approximants are rational functions, we concentrate on a single component of $v(\omega)$, say $v^{j}(\omega)$, and we write its power series representation in the form:

$$
\begin{equation*}
v^{j}(\omega)=\sum_{i=0}^{\infty} v_{i}^{j}\left(\omega-\omega_{0}\right)^{i} \tag{18}
\end{equation*}
$$

where the coefficient $v_{i}^{j}$ is scalar. A Pade approximant of the power series equation (18) is a rational function of the form:

$$
\begin{equation*}
[N / M](\omega)=\frac{Q_{N}(\omega)}{P_{M}(\omega)} \tag{19}
\end{equation*}
$$

where

$$
Q_{N}(\omega)=\sum_{i=0}^{N} q_{i}\left(\omega-\omega_{0}\right)^{i} \text { et }
$$

$$
\begin{equation*}
P_{M}(\omega)=\sum_{i=0}^{M} p_{i}\left(\omega-\omega_{0}\right)^{i} \tag{20}
\end{equation*}
$$

We take $p_{0}=1$, the $M+N+1$ unknowns can be obtained by the condition that the equation

$$
\begin{equation*}
v^{j}(\omega) \approx[N / M](\omega) \tag{21}
\end{equation*}
$$

holds up to terms $O\left(v^{N+M+1}\right)$. This equation implies that

$$
\begin{equation*}
Q_{N}(\omega)=P_{M}(\omega) \sum_{i=0}^{\infty} v_{i}^{j}\left(\omega-\omega_{0}\right)^{i} \tag{22}
\end{equation*}
$$

Then we have

$$
\begin{equation*}
\sum_{i=0}^{N} q_{i}\left(\omega-\omega_{0}\right)^{i}=\sum_{i=0}^{\infty} l_{i}\left(\omega-\omega_{0}\right)^{i} \tag{23}
\end{equation*}
$$

where

$$
\begin{equation*}
l_{i}=\sum_{k=0}^{i} v_{i-k}^{j} p_{k} \tag{24}
\end{equation*}
$$

Hence $q_{i}$ and $p_{i}$ can be determined from the following system

$$
\begin{align*}
& p_{0}=1  \tag{25a}\\
& q_{i}=\sum_{k=0}^{i} v_{i-k}^{j} q_{k}, \quad \text { if } 1 \leq i \leq N  \tag{25b}\\
& p_{i}=0 \quad, \quad \text { if } i>N  \tag{25c}\\
& \sum_{k=1}^{i} v_{i-k}^{j} p_{k}=-p_{0} v_{i}^{j}, \text { if } N<i<N+M  \tag{25d}\\
& p_{i}=0, \text { if } i>M \tag{25e}
\end{align*}
$$

Hence the unknown coefficients of the Pade approximant can be determined from linear system. We use the diagonal Padé approximation ( $N=M$ ) which is more accurate; in this case we have $2 N+1$ unknown coefficients.

## VI. Numerical results

Figure 2 shows a typical mesh used in the computation for the loop and the surrounding air. The electromagnetic analysis was performed over a broad band $\mathrm{I}=[0 \mathrm{GHz}, 20 \mathrm{GHz}]$. In a first step, the studied frequency band is divided in $\mathrm{L}=4$ intervals $\mathrm{I}_{\mathrm{i}}, \mathrm{i}=1, . ., \mathrm{L}$ such that $\mathrm{I}_{1}=[0 \mathrm{GHz}, 5 \mathrm{GHz}], \mathrm{I}_{2}=[5 \mathrm{GHz}$, $10 \mathrm{GHz}], \mathrm{I}_{3}=[10 \mathrm{GHz}, 15 \mathrm{GHz}]$ and $\mathrm{I}_{4}=[15 \mathrm{GHz}, 20$ GHz ]. In each band the centre frequency has been chosen in the middle.

$$
\text { We denote }[N / M]_{i}(\omega), i=1, . ., L \text { the }
$$

Padé approximation of the impedance in each
interval $\mathrm{I}_{\mathrm{i}}$. In this case $N=M=2$. The impedance can be written as:

$$
\begin{equation*}
Z(\omega)=\sum_{j=1}^{L} Z^{j}(\omega) \tag{26}
\end{equation*}
$$

where

$$
Z^{j}(\omega)=\left\{\begin{array}{cccc}
{[2 / 2]_{j}(\omega)} & \text { if } & \omega & \in
\end{array} I_{j} .\right.
$$



Fig. 2. The volume of the computation.
The comparison between a standard finite element model and the Padé approximation is shown on Fig. 3 and Fig. 4. The two curves are in an excellent agreement over the whole wide frequency band. In the standard approach the finite element problem has been solved for a number of 200 frequencies to obtain the behaviour of the curve. With the Padé approximation only 4 frequencies are needed. The computational cost is then significantly reduced since the amount of time required to find the Padé coefficients is negligible.


Fig. 3. Argument of the impedance: The comparison between 3D model and the Pade approximation [2/2] over four bands.

For three bands it was shown that the Padé approximant [3/3] gives very good results. Thus, for two bands a [3/3] approximation gives a bad
approximation at the junction between the two bands Fig. 5. In this case an increase of the order of the Padé approximant is required: we show the corresponding results for a [5/5] approximation in Fig. 6.


Fig. 4. The Amplitude: The comparison between 3D model and the Pade approximation [2/2] over four bands.


Fig. 5. Argument of the impedance: The comparison between 3D model and the pade approximation [3/3] over two bands.

## VII. Conclusion

This paper describes an accurate and very efficient FEM approach to compute the impedance of an antenna over a wide frequency band. The electromagnetic problem addressed in this work is a fully three-dimensional one. In a standard finite element approach the linear system has to be solved for each frequency of interest. The power series expansion method presented in this work resolves this problem very efficiently. The advantage of this approach is that only a single resolution of the linear system is required to evaluate the series expansion using a Padé approximation. Then the approach
allows one to cover a whole given frequency band with a minimum number of resolution of the linear system. In particular the resonance peaks are very well recovered. It is worth to be noted that satisfactory results are obtained when using a SilverMüller radiation condition: the impedance is computed from the near-field and numerical values near the antenna are not very sensitive to the boundary condition. Work is in progress to combine a Padé approximation with PML (perfectly matched layer) in order to address more general radiation problems in three dimensions for which global quantities have to be obtained over a wide band.

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# EBG Design using FSS Elements in Rectangular Waveguide 

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#### Abstract

A novel waveguide based EBG structure is originated by periodically loading FSS strip elements in rectangular waveguide. Efficient and accurate Hybrid MoM-Immiittance Approach is used for the full-wave characterization, which is validated by experimental results. A parametric study of effect of various factors on the EBG width has been done. Various existing and novel FSS strips have been investigated to improve the roll-off characteristics in the passband. Double square loop FSS strip loaded waveguide gives improvement in the roll-off factors. Such novel waveguide based EBG structures may be used in the design of harmonic suppressed waveguide filters, band reject filters and suppression of harmonics for waveguide resonators or antennas.


## I. INTRODUCTION

Photonic bandgap (PBG) structures are artificially made structures with periodically loaded obstacles in 1D, 2-D, or 3-D. They are capable of forbidding electromagnetic propagation in either all or selected directions [1]-[3]. Although periodic structures have been investigated in microwave community for many decades, new ideas and concepts developed in optical domain [4][5] have renewed interest in microwave area. EM waves behave in such crystals similar to that of electronic behavior in semiconductors hence it is also named as Electromagnetic bandgap (EBG) structure. In microwave community, preferable nomenclature is EBG structure [6]. As in Photonic crystals (PC), photon propagation is impeded by periodic discontinuity, EM waves in EBG materials are hindered by periodic discontinuity making it a slow wave structure. The slow wave behavior in passband characteristics of EBG structures can be used as a slow wave medium for size miniaturized microwave devices and circuits [7]. Surface waves propagating in high dielectric constant slabs carry substantial energy in unwanted directions and create unnecessary coupling between the devices. EBG structures can used to alleviate these problems by suppressing higher order modes and surface waves [8]. In filters, EBG structures are employed for harmonic suppression and improving filter performance without increasing dimension of the device [9]. In antenna design, EBG structures can be used to enhance antenna broadside gain, to suppress surface
waves and to reduce cross-polarization levels [10]. On the basis of dimensions in which periodic perturbations like dielectric rods, holes and patterns in waveguides and microstrip substrates are introduced, EBG structures can be categorized as 1-D, 2-D or 3-D. Conventional EBG structures are 1-D/2-D/3-D periodic structures that satisfy Bragg's conditions, i.e, inter-cell separation (period) is close to half guided wavelength. Frequency Selective Surface (FSS) is widely used in microwave and optical engineering as spatial and frequency filters [11]-[12]. In this paper, we have originated an alternative 1-D waveguide based EBG structure by periodically loading rectangular waveguide with FSS strip elements printed on dielectric substrate. Efficient and accurate Hybrid MoM-Immittance Approach [13] has been used for all the simulation works.

## II. ORGANIZATION OF THE PAPER

First, a wide resonant strip in X-band waveguide is validated with experimental results. There is good agreement between the Hybrid MoM-Immittance and experimental results. Next, we study the effect of various parameters on the EBG width for simple square FSS strip loaded waveguide. From the parametric study of effect of various factors on the EBG width, we have designed an optimized wideband EBG structure using the FSS square strip. It has been observed that although square FSS strip loaded periodic waveguide structure gives a very broad EBG width, the roll-off characteristics in the passband of the periodic waveguide structure is not good. Hence we try to improve this performance by considering various existing FSS and novel FSS strip elements loaded waveguide structure. Seven FSS strip structures viz., square (FSS1), square loop (FSS2), ring (FSS3), cross (FSS4) and other novel FSS structures: double square loop (FSS5), FSS6 (FSS2+FSS4) and FSS7 (FSS1+FSS4) loaded waveguide has been investigated. Double square loop loaded periodic waveguide structure shows a promising candidate for improving the roll-off factors in the waveguide based EBG structures. The scattering performance for a periodic waveguide based EBG structure loaded with different number of double square loop unit/cells have been investigated. It is observed that with the increase of number of unit/cells,
insertion loss goes into deep rejection band and roll-off factor in the passband improves. Such waveguide based EBG structures can be used for various applications like design of harmonic suppressed waveguide filters, band rejection filters and harmonic suppression of waveguide resonators.

## III. EXPERIMENTAL VALIDATION OF HYBRID MOM-IMMITTANCE APPROACH

Hybrid MoM-Immittance approach is the hybrid of MoM and Immittance approach [13]. In this method, Galerkin's MoM method and Fourier transform techniques are employed to transform the electric-field integral equation (EFIE) into a matrix system of linear equations. Dyadic Greens' functions are calculated from the TE and TM circuit models for 1-D inhomogeneous multilayered structures. It has been employed for study of guided-wave characteristics of printed periodic waveguide structures [14]. This method has been validated for various waveguide based structures in comparison with analytical results [14] and HFSS simulation results [13]-[14]. Here we will do an experimental validation of this efficient and accurate approach. Fig. 1(a) shows a X-band waveguide with a centered strip of width $=0.280$ inch and depth $=0.360$ inch. The equivalent circuit of the waveguide structure under investigation can be represented by a shunt susceptance as shown in Fig. 1(b). Fig. 1(c) illustrates the normalized susceptance versus frequency. Note the choice of basis functions: half-basis functions are employed at the edge where the strip touches the waveguide walls. It can be observed that there is close agreement between the Hybrid MoM-Immittance Approach and experimental results from [15].


Fig. 1. A rectangular waveguide with a wide resonant strip (a) Cross section, (b) Equivalent circuit, and (c) Normalized susceptance of a centered strip of $w=0.280$ inch and $\mathrm{d}=0.360$ inch.


Fig. 2. 3-D geometry of a rectangular waveguide $(\mathrm{a} \times \mathrm{b})$ loaded with a square strip (l) printed on a dielectric layer of thickness h (3 unit cell).


Fig. 3. Insertion loss versus frequency for different dimension of square strip (1).

## IV. EFFECT OF VARIOUS PARAMETERS ON EBG WIDTH

Figure 2 shows a rectangular waveguide loaded with three transverse layers of FSS square strips printed on dielectric layer of thickness $h$. In order to understand the EBG performances properly, let us investigate the effect of various parameters on the EBG width. The actual waveguide based EBG structure composed of periodic waveguide structures with many transversal layers of FSS strips. Here, we have considered a three unit/cell finite periodic waveguide structure for investigation on the -10 dB insertion loss EBG width. The various parameters which may control the EBG performances are:

1) Dimension of FSS square strip (1),
2) Dielectric constant of dielectric layer on which the FSS elements are printed $\left(\varepsilon_{\mathrm{r}}\right)$,
3) Thickness of the dielectric layer (h),
4) Period of the periodic waveguide structure (p),
5) Number of unit/cells (h).

The square FSS strip elements are printed on a dielectric layer of $\varepsilon_{\mathrm{r}}=3$ and thickness $\mathrm{h}=1 \mathrm{~mm}$. The waveguide dimensions are $\mathrm{a}=22.86 \mathrm{~mm}$ and $\mathrm{b}=10.16$ mm and the dimension of the FSS square strip elements is chosen as $1=7 \mathrm{~mm}$.

## A. Dimension of FSS Square Strip (l)

It has been observed that with increase of dimension for the FSS square strip from $1=7 \mathrm{~mm}, 8 \mathrm{~mm}$, and 9 mm (other parameters kept the same), there is downward shift in the resonant frequency of the FSS square strip element as illustrated in Fig. 3. There is also significant increase in the -10 dB insertion loss EBG width with increase in the dimension of FSS square strip (refer to Table I).

## B. Dielectric Constant of Dielectric on which the FSS Elements are Printed ( $\varepsilon_{r}$ )

As the relative permittivity $\left(\varepsilon_{\mathrm{r}}\right)$ of the dielectric layer on which FSS square strip are printed increases from 3, 4, and 5.7 (other parameters kept constant for all the three cases), the resonant frequency of FSS square printed waveguide structure decreases as depicted in Fig. 4 and the fractional EBG width increases as tabulated in Table I.

## C. Thickness of the Dielectric Layer ( $h$ )

An interesting observation is that when we increase the thickness of the dielectric layer, the fractional EBG width decreases. For a dielectric layer of $\varepsilon_{\mathrm{r}}=3$ and thickness of the dielectric layer $\mathrm{h}=1 \mathrm{~mm}$, the fractional EBG width is $24 \%$ (refer to Fig. 5 and Table I). It reduces to $13.51 \%$ as we increase the dielectric layer thickness h to 3 mm , keeping same the other parameters of the waveguide structure of Fig. 2.

## D. Periodicity (p)

It is a good idea to investigate the role of periodicity p in increasing the EBG width. Fig. 6 illustrates the bandstop of square FSS printed waveguide structure of Fig. 2 for various periodicity $p$. It can be observed that there is a downward frequency shift in bandstop as periodicity p is increased from $\mathrm{p}=4.8 \mathrm{~mm}, 5.8 \mathrm{~mm}$, and 6.8 mm . Besides, there is visible enhanced bandwidth for periodicity of $\mathrm{p}=4.8 \mathrm{~mm}$ in comparison to other values of $p$. It is because the connecting waveguide section approaches half guided-wavelength, the frequency at which each square loop resonates, thereby further widening the EBG width. The scattering parameter results shown are for two unit/cells i.e., $\mathrm{n}=2$ for periodic waveguide structure.


Fig. 4. Insertion loss versus frequency for different values of relative electrical permittivity $\left(\varepsilon_{\mathrm{r}}\right)$.

## E. Number of Unit/cells (n)

Fig. 7 shows insertion loss $\left(\mathrm{S}_{21}\right)$ for the finitely extended periodic structure with number of unit cells varying from $\mathrm{n}=2,3$, and 5 for a fixed periodicity $\mathrm{p}=$ 4.8 mm . It can be observed that as number of unit cells (n) increases, insertion loss goes into deep rejection band as mentioned in many literatures [3]. There is also slight decrease in the EBG width as the number of unit/cells increases as shown in Fig. 7.

Table I. -10 dB insertion loss EBG width versus various parameters.

| a | 9 mm | 8 mm | 7 mm |
| :---: | :---: | :---: | :---: |
| EBG width | $(18.4-8.0) / 13.2$ | $(18.2-12.2) / 15.2$ | $(18.2-14.3) / 16.2$ |
|  | $78.78 \%$ | $39.47 \%$ | $24 \%$ |
| Relative <br> permittivity | 3 | 4 | 7 |
| EBG width | $(18.2-14.3) / 16.25$ | $(16.7-12.7) / 14.7$ | $(14.8-10.8) / 12.8$ |
|  | $24 \%$ | $27.2 \%$ | $31.25 \%$ |
| h | 1 mm | 2 mm | 3 mm |
| EBG width | $(18.2-14.3) / 16.25$ | $(16.4-13.8) / 15.1$ | $(15.8-13.8) / 14.8$ |
|  | $24 \%$ | $17.21 \%$ | $13.51 \%$ |
| p | 4.8 mm | 5.8 mm | 6.8 mm |
| EBG width | $(15.5-9.6) / 12.55$ | $(15.3-10.1) / 12.7$ | $(15.2-10.8) / 13 . \mathrm{C}$ |
|  | $47.01 \%$ | $40.94 \%$ | $33.84 \%$ |



Fig. 5. Insertion loss versus frequency for different thickness of the dielectric layer (h).


Fig. 6. Insertion loss versus frequency for different period (p).


Fig. 7. Insertion loss versus frequency for different number of units/cells (n).

## V. COMPACT WAVEGUIDE BASED EBG STRUCTURE

A waveguide based EBG structure is constructed by periodically loading FSS square strips in rectangular waveguide whose geometry (side view) is shown in Fig. 7 (b) and its front view is depicted in Fig. 7 (a). The thickness of the dielectric layer $h$ is chosen as 1 m and the relative permittivity of the dielectric layer is 3 . The period p is chosen approximately $\lambda_{g} / 2$ (at the Bragg's frequency) which is equal to 4.8 mm . The scattering performance of the designed EBG structure is plotted in Fig. 7 (c). It can be observed that there exists a deep bandgap or forbidden band with $\left|\mathrm{s}_{11}\right|$ of about 0 dB in the frequency region from 10.5 GHz to 15.5 GHz . The EBG structure is compact because of the increased slow-wave factor due to the dielectric layer. For $\mathrm{n}=5$, the overall length of the 1-D EBG structure is $5 \mathrm{~mm} \times 4.8 \mathrm{~mm}=24$ mm only.


Fig. 8. Geometry of the waveguide based EBG structure; (a) Front view, (b) Side view, and (c) Scattering performance of the waveguide based EBG structure.

## VI. IMPROVING THE ROLL-OFF CHARACTERISTICS

In this section, we will consider various FSS strip loaded waveguide structure to improve the EBG roll-off characteristics in the passband. Several FSS strip structures viz., square (FSS1), square loop (FSS2), ring (FSS3), cross (FSS4) and other novel FSS structures: double square loop (FSS5), FSS6 (FSS2+FSS4) and FSS7 (FSS1+FSS4) loaded waveguide have been investigated and their front view is depicted in Fig. 9 (a). The insertion loss in dB for first 4 FSS structures loaded waveguide structure is shown in Fig. 9(b). The insertion loss in dB for remaining 3 novel FSS structures loaded waveguide structure is depicted in Fig. 9(c). It can be observed that the insertion loss characteristics after resonance go upward towards zero as the frequency increases whereas the insertion loss before resonance usually touches around -10 dB and goes downward further as the frequency decreases. To improve the rolloff characteristics, it should go to 0 dB instead of -10 dB and then should go down as the frequency decreases. The best insertion loss characteristics for improving the EBG roll-off characteristics is for FSS 5 i.e., double square loop FSS structure.

Since the double square loop FSS strip loaded waveguide unit/cell gives the best roll-off characteristics. Let us study the effect of number of unit/cells on the EBG performance specially the roll-off characteristics. From Fig. 10 (b), we can see that as the number of unit/cells of the finite periodic waveguide structure whose geometry depicted in Fig. 10 (a), the roll-off factor increases. The dimensions of the double square loops are chosen as $1_{1}=9 \mathrm{~mm}$ and $1_{2}=5 \mathrm{~mm}$. Both the square loops have strip thickness 1 mm . They are transversely put in a X-band waveguide. The period p is chosen as 5.58 mm and thickness of the dielectric layer of dielectric constant 3 is taken as $\mathrm{h}=1 \mathrm{~mm}$.

(a)

(b)

(c)

Fig. 9. (a) Front view of the waveguide loaded with various FSS strips, Insertion loss for (b) first 4 FSS strip elements, and (c) remaining 3 novel FSS strip elements.


Fig. 10. (a) Geometry of double square loop FSS printed finite periodic waveguide structure acting as EBG structures and (b) Magnitude of scattering parameters.

## VII. CONCLUSION

In this paper, we have originated and investigated a novel EBG structure by periodically loading transversally FSS strip elements in rectangular waveguide. First, we have done a parametric study on effect of various factors on the EBG width. Based on the parametric study, an optimized wideband waveguide based EBG structure using the square FSS strip element has been designed.

Although, the EBG structure using simple square FSS strip elements exhibits broad EBG width, the roll-off factor in the passband is not good. We have investigated various existing and novel FSS strip elements to improve its roll-off factors. Double square loop FSS printed waveguide structure shows a promising candidate for this. It has also been observed that with the increase of number of unit/cells, the insertion loss goes into deep rejection band and the roll-off factor in the passband improves. Such EBG structures can be used in the design of harmonic suppressed waveguide filters, band rejection filters and harmonic suppression of waveguide resonators.

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# Electromagnetic Modeling and Design of Dual-Band Septum Polarizers 

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#### Abstract

A highly efficient, full-wave electromagnetic model and CAD of septum polarizers terminated with a square or circular output waveguide is presented. The model is based on the mode-matching and generalized S-matrix techniques. The capabilities of the CAD program are demonstrated by the design of dual-band polarizers where the separation between two bands is more than $50 \%$. For a square output polarizer, results are compared and found to be in good agreement with data obtained with the WASP-NET program. A discussion on the inherent limitations of the achievable performance and critical parameters is presented. An influence of higher-order modes propagating in the polarizer output over the high-frequency operation band on the radiation and polarization patterns of a septum polarizer combined with a corrugated conical horn is specially investigated.


## I. INTRODUCTION

Septum polarizers (SP), built on a square waveguide with stepped ridged waveguide junctions, have found many applications in antenna and other microwave systems. Single-band configurations with a square output waveguide were widely investigated. The fourstep polarizer, initially designed in [1] by trial and error experimental methods, satisfied the return loss criteria but required an additional dielectric-slab phase shifter to adjust the 90 -degree phase difference between the $\mathrm{TE}_{10}$ and $\mathrm{TE}_{01}$ orthogonal modes. There were difficulties in producing an acceptable phase shift with an experimental tuning of the polarizer without phaseadjusting structures, [2].

Full-wave solutions and numerical optimization procedures allowed the design of various single-band polarizers with a square output port that met the required characteristics over a broad bandwidth. The design results of a five-step septum polarizer without an
additional phase adjustment were reported in [3]. A section of corrugated waveguide was proposed to use as the phase-correction device, [4]. The broadband polarizers with a stepped-thickness septum were investigated in [5], [6]. A dynamical optimization procedure that included in the optimization process not only the stepped septum configuration but also several step discontinuities (placed before and after the septum) was proposed in [7]. An algorithm based on the approximation of aperture field distribution by Gegenbauer polynomials was used in [8] to analyze the SP experimentally studied in [6].

The septum polarizer with a circular output port is more convenient for antenna applications because it can be connected directly to a flange of smooth or corrugated conical horn. An example of the design of a square septum polarizer, which is directly combined with a smooth conical circular waveguide horn, is shown in [9]. A design procedure for the single-band polarizer with a circular output waveguide is discussed in [8] however results of this design were not presented. Some results are reported in [10] for compact three- and four-step polarizers.

To the authors' knowledge, except for [11], dual-band septum polarizers operating over two distant (with more than $15 \%$ separation) frequency bands have not been yet considered in the literature. In addition to the need to meet common requirements (such as return loss, isolation, and axial ratio performance), in this case a new parameter has to be considered: the level of suppression of the higher-order modes. These modes are always excited and can propagate over the highfrequency band. In the case of square output port, these modes are the $\mathrm{TE}_{11}$ and $\mathrm{TM}_{11}$ ones, and in the case of circular output port they are the axial-symmetric $\mathrm{TM}_{01}$ and two polarization-degenerated $\mathrm{TE}_{21}$ modes.

In the present paper, the electromagnetic model, optimization procedure, designing results for the dualband polarizers with square or circular output waveguides, and some features of their characteristics are discussed.

## II. DESIGN PROCEDURE

## II.1. The Electromagnetic Model

A polarizer with a circular output waveguide and its geometrical parameters are shown in Fig. 1. The electromagnetic model is based on the generalized $S$ matrix technique. In applying this technique, the keyelements are identified with the following waveguide discontinuities:

- double step in a rectangular waveguide;
- bifurcated-to-ridged waveguide transition;
- ridged-to-ridged waveguide junction;
- square-to-square waveguide junction;
- square-to-circular waveguide transition.

The full-wave $S$-matrices of all the key-elements are calculated using the mode-matching technique. In the case when the jointed circular and square waveguides have overlapping cross-sections (as shown in Fig. 1(a)), this transition is considered as two waveguides connected via a zero-length square waveguide with the wall size $a_{v i r t}=D$ where $D$ is the diameter of the output circular waveguide. In this case, the circular output is circumscribed to the virtual square waveguide. All the aforementioned key-elements are calculated and combined with the aid of an electromagnetic solver similar to the one reported in [12]. The eigen-value problem for the single-ridged waveguide is solved using the moment method with basis and test functions that take into account the field behavior near edges. Some details of this method can be found in [13].

## II.2. The Optimization Procedure

The SP-CAD program allows the design of a dual- and single-band polarizer according to a given set of specifications such as:

- return loss ( $R L$ );
- isolation (IS) between input waveguides;
- axial ratio $(A R)$ of the resulting field generated by two outgoing orthogonal dominant modes;
- suppression level $(S L)$ of the higher parasitic modes in the output port if the latter is overmoded. The used optimization procedure is based on the descent method.

The SP-CAD program operates with the given number
of polarizer components. Some of them are fixed, and others are changed during the optimization process. For a given number of septum steps $N$, the fixed geometrical parameters are:


Fig. 1. Schematic structure of a three-step polarizer with a circular output waveguide. (a) General view, (b) Longitudinal-section view.

- the cross-sections of the two identical input rectangular waveguides, $a_{i n p} \times b_{i n p}$;
- the septum thickness, $t$;
- the size of a square housing for the ridged waveguide sections, $a_{s q} \times a_{s q}$;
- the size of the transformer square section, $a_{t r} \times a_{t r}$, placed between the above-mentioned housing and output circular waveguide;
- the diameter of the circular waveguide, $D$.

The other available SP configurations have the square output port with $a_{o u t}=a_{t r}$.
Generally, the gap sizes $w_{i}$ and lengths $l_{i}, i=1,2, \ldots, N$ of the single-ridged waveguide sections, the length $l_{r w}$ of the two identical rectangular waveguides of the $a_{i n p} \times\left(a_{s q}-t\right) / 2$ cross section, and the lengths $l_{s q}$ and $l_{t r}$ of the square waveguide sections (see Fig. 1(b)) are included in the vector of objective variables for the
optimization procedure. The following error function is minimized during the optimization:

$$
\begin{align*}
F(\vec{x})=\min \sum_{j=1}^{2} & \sum_{m=1}^{M_{j}}\left[W_{R L}^{(j)}\left(\frac{R L_{j}}{R L_{m}^{(j)}}\right)^{2}+W_{I S}^{(j)}\left(\frac{I S_{j}}{I S_{m}^{(j)}}\right)^{2}\right. \\
+ & \left.W_{A R}^{(j)}\left(\frac{A R_{m}^{(j)}}{A R_{j}}\right)^{2}+W_{S L}^{(j)}\left(\frac{S L_{j}}{S L_{m}^{(j)}}\right)^{2}\right] \tag{1}
\end{align*}
$$

where
$-\vec{x}$ is the vector of the objective variables;

- $M_{j}$ is the given number of the frequency points $f_{m}^{(j)}$ within the $j$ th specified band;
- $W_{R L}^{(j)}$ is the return loss weighting coefficient;
$-R L_{j}$ is the specified return loss value (in dB );
$-R L_{m}^{(j)}=R L_{m}^{(j)}\left(f_{m}^{(j)}, \vec{x}\right)$ is the actual return loss value calculated at the frequency $f_{m}^{(j)}$ and with the current values of the objective variables.
The similar notations are used in (1) for the other controlled characteristics.


## II.3. The Initial Guess

To the authors' knowledge, there are no well established models that can be applied for the preliminary SP synthesis. As in any multivariable optimization routine based on gradient or descent methods, the selection of the initial guess is the most difficult problem. Various approaches can be adopted to overcome this problem. The first of them consists in the electrical scaling of a known SP geometry (for example, from [1] to a new frequency operation band. This approach can be used for a simple configuration representing a stepped septum in a straight square waveguide (as in [1], [3]).

With the SP-CAD, the choice of the invariable polarizer dimensions is based on the following considerations. The square housing size $a_{s q}$ must be such that the cutoff frequency of the $\mathrm{TE}_{11}$ and $\mathrm{TM}_{11}$ modes in the hollow $a_{s q} \times a_{s q}$ waveguide is between the polarizer operation bands. Preferable size $a_{\text {out }}$ for the SP with a square output waveguide is that permits to propagate the higher $\mathrm{TE}_{11}$ and $\mathrm{TM}_{11}$ modes over the highfrequency band only.

If the SP with a circular output is required, the output diameter $D$ can be chosen using similar considerations. In the last case, the recommended size of square
transformer section $a_{t r}$ is such that the cutoff frequencies of dominant modes in the circular and transformer waveguide section are close to each other. This takes place if $a_{t r} \approx 0.85 D$.

A simple way to define the initial septum geometry is to define the ridge gap dimensions according to the linear representation (for example) such as $w_{i}=a_{s q}(1-i /(N+1)), i=1,2 \ldots N$. The lengths of the corresponding waveguide sections can be set as $l_{i}=(0.4-0.5) a_{s q}$.

## II.4. The Surrogate Models

The SP-CAD allows two options: the optimization can be done either with the exact numerical models of all components or with interpolated models of the generalized $S$-matrices of some components (called here "surrogate" models).

Before starting the optimization process, onedimensional frequency buffers are created for the exact $S$-matrices of the polarizer components with fixed geometry. For example, for a square-to-circular waveguide transition, the buffer is calculated at the given set of frequency points $f_{m}^{(j)}$ only once, and is not recalculated during the optimization process.

The exact models of polarizer components with varying geometry (such as the ridged-to-ridged waveguide junctions) are substituted by their two-dimensional surrogates. The latter are obtained by the interpolation of the full-wave $S$-matrices of the above-mentioned junctions at a given set of sampling points for geometrical parameters at each frequency $f_{m}^{(j)}$. In this case, the varied geometrical parameters are the sizes of gaps $w_{n}, n=1,2, \ldots, N$ of adjacent ridged waveguide sections. The calculation of the database for the surrogate models is a time-consuming procedure, but once the database is created, the overall CPU time used for the polarizer optimization is considerably reduced. The database is stored in files and can be used repeatedly to optimize polarizers differing only by the parameters $a_{t r}$ and/or $D$.

The program can operate in various accuracy modes. The low-accuracy mode with using the surrogate models allows a rough evaluation of a certain SP topology. This mode is a fast and suitable way for the definition of the initial guess for the final optimization performed in the high-accuracy mode. The frequency


Fig. 2. Performance of the designed four-step WR-90 septum polarizer with a square output waveguide and comparison with the WASP-NET analysis. Dimensions (in millimeters): $a_{i n p}=22.86, b_{i n p}=10.16, a_{s q}=20.8, l_{r w}$ $=5.39, t=1.5, w_{1}=17.23, l_{1}=9.03, w_{2}=13.85, l_{2}=$ $9.53, w_{3}=10.48, l_{3}=8.46, w_{4}=4.83, l_{4}=3.72, a_{\text {out }}=$ 20.8.
analysis of the optimized SP is performed with using the exact numerical models of all the SP components in the chosen accuracy mode.

## III. SOME RESULTS OF THE SP DESIGN

The SP-CAD program has been successfully applied in the design of polarizers having a square or circular output waveguide. The results for four-step optimized polarizers operating in several frequency ranges are presented in Figs. 2 to 5. They have been analyzed with the high-accuracy exact model under the SP dimensions shown in the figures captions. The SP operation bands are delimited in the figures by the vertical dasheddotted lines. Figures marked as "(b)" show the axial ratio $(A X)$, return loss $(R L)$, and isolation (IS) polarizers performance whereas the "(b)" figures characterize the relative powers (suppression levels $(S L)$ ) of unwanted


Fig. 3. Performance of the designed four-step WR-90 septum polarizer with a circular output waveguide. Dimensions (in millimeters): $a_{\text {inp }}=22.86, b_{\text {inp }}=10.16$, $a_{s q}=20.5, l_{r w}=4.1, t=1.5, w_{1}=17.11, l_{1}=9.77, w_{2}=$ 14.03, $l_{2}=10.16, w_{3}=10.68, l_{3}=8.35, w_{4}=4.57, l_{4}=$ $3.45, l_{s q}=5.8, a_{t r}=21.6, l_{t r}=8.48, D=24.4$.
higher-order modes in the SP output.
The optimization of all the SPs has been carried out with the following specification within two bands:

- return loss $R L \leq-20 \mathrm{~dB}$;
- isolation between the input waveguides $I S \leq-25$ dB;
- axial ratio (calculated from the dominant modes amplitudes in the SP output) $A R \geq-1 \mathrm{~dB}$;
- higher-order mode suppression level $S L \leq-20 \mathrm{~dB}$.

Figs. 2(a)-(b) show the predicted performances of the optimized square-output polarizer designed for two bands, $8.3 \mathrm{GHz}-9.4 \mathrm{GHz}$ and $11.3 \mathrm{GHz}-12.4 \mathrm{GHz}$. Input ports are the WR-90 waveguides. The resulting characteristics are close to the specified ones within both bands. For comparison purposes, the numerical data obtained with WASP-NET program, described in
[14], and [15], are plotted by symbols in Figs. 2(a)-(b) as well. Good agreement between data obtained with two programs is evident. This is mainly due to the fact that both programs are based on the mode-matching technique and the calculations were carried out with the same frequency, $f_{\max }=72 \mathrm{GHz}$, limiting maximal cutoff frequencies of modes in ports of all the keyelements. Some differences at high frequencies are due to the following reasons. First, two programs uses the different algorithms in the calculation of the ridged waveguide modal basis. In addition, the number of modes taken into account between any connected discontinuities was set as twenty in the SP-CAD program when using the $S$-matrix technique. In the WASP program, this number is determined indirectly: all modes in connecting waveguide sections with cutoffs $f_{\text {con }} \leq f_{\max }$ are taken into account. The WASP results in Fig. 2 were calculated at $f_{\text {con }}=0.55 f_{\text {max }}$.

A characteristic feature of the frequency responses shown in Fig. 2(a) is a sharp resonance at $f=9.88 \mathrm{GHz}$. It shows up before the cutoff frequency of the $\mathrm{TE}_{11}$ and $\mathrm{TM}_{11}$ modes $\left(f_{\text {cut }} \approx 10.19 \mathrm{GHz}\right)$ in the output waveguide (see Fig. 2(b)). When passing this frequency point, the $\mathrm{TE}_{10}$ mode transmission coefficient and axial ratio response decrease drastically and the differential phase shift exhibits a sharp jump. It is the resonance effect that limits the widths of the polarizer operation bands. This effect is due to the resonance of the first higher quasi- $\mathrm{TE}_{11}$ mode that propagates in the ridged waveguide sections and does not propagate in the input and output waveguides. Similar resonances are named as resonances on higher "ghost" (or "closed") modes and have been studied in the literature (see, for example, [16], [17]). The discussed resonance has been experimentally observed in [3] and specially noted in [5] when analyzing single-band SPs. It is an inherent feature of the SPs and cannot be avoided. One can only control the resonance frequency value by a proper choice of the SP size $a_{s q}$.

The results for the optimized WR-90 SP with a circular output waveguide are shown in Fig. 3. The return loss, isolation, and axial ratio responses meet the specification over the two specified bands practically (see Fig. 3 (a)). The circular output diameter for this SP is chosen so that the parasitic $\mathrm{TM}_{01}$ mode cutoff ( $f_{\text {cut }}=9.41 \mathrm{GHz}$ ) is outside the low-frequency band and the $\mathrm{TE}_{21}$ cutoff ( $f_{\text {cut }}=11.94 \mathrm{GHz}$ ) is within the high-frequency band. The curves of the parasitic mode powers are shown in Fig. 3(b). There are two $\mathrm{TE}_{21}$
modes, $\mathrm{TE}_{21}^{(s)}$ and $\mathrm{TE}_{21}^{(c)}$, which transversal field components are orthogonal and rotated by $45^{\circ}$ relative to each other. Here, the superscript $s$ or $c$ denotes the $\sin n \varphi$ or $\cos n \varphi$ polar-angle function, respectively, in the Herz vector representation for the modes in a circular waveguide. It is remarkable that the level of the $\mathrm{TE}_{21}^{(s)}$ mode is considerably higher than the $\mathrm{TE}_{21}^{(c)}$ one. This is caused by that the $\mathrm{TE}_{21}^{(s)}$ mode is efficiently excited by the $\mathrm{TE}_{11}$ mode propagating in the SP square waveguide sections because both these modes have similar field distributions.

In contrast to the polarizer with a square output, the frequency responses in Fig. 3 (a) have two resonances. The first of them has the same nature as that in Fig. 2 (a). The second resonance appears before the beginning of high-frequency band. It is caused by the transformer square section $\mathrm{TE}_{11}$ mode coupled with the $\mathrm{TE}_{21}^{(s)}$ mode in the circular output waveguide and playing a role of that closed in the SP cavity. This resonance is a reason of an increased level of the $\mathrm{TM}_{01}$ mode before the highfrequency band (see Fig. 3 (b)). To shift the resonance to a lower frequency, a larger output diameter has to be chosen. However, in this case the $\mathrm{TM}_{01}$ mode starts to propagate within the low-frequency band that is not acceptable for some SP applications. A similar resonance peak does not appear before the $\mathrm{TM}_{01}$ mode cutoff. This mode is excited by the $\mathrm{TM}_{11}$ mode of the transformer square section with the cutoff frequency higher than the $\mathrm{TM}_{01}$ one. In this case, there are no closed modes and a resonance has not to be appeared, [17]. The aforementioned resonances can lead to some difficulties in designing the SP with a circular output port at a smaller separation between two bands.

The separation between the bands of the aforementioned polarizers is $46 \%$ with respect to the total SP operation range and both these bands are within the WR-90 waveguide frequency range. Figs. 4 and 5 demonstrate the SP-CAD capabilities in designing the millimeter-wave polarizers provided with the input WR-34 rectangular waveguides and operating over the bands $20 \mathrm{GHz}-21.5 \mathrm{GHz}$ and $29.5 \mathrm{GHz}-31.5$ GHz . The SPs have been designed under the performance specification identical to that for the WR90 polarizers. The separation between the bands already is about $70 \%$. It should be noted that the low-frequency SP band is entirely outside of the standard frequency range ( $22 \mathrm{GHz}-33 \mathrm{GHz}$ ) of the WR-34 waveguide.

The square-output SP performance shown in Figs. 4


Fig. 4. Performance of the designed four-step WR-34 septum polarizer with a square output waveguide. Dimensions (in millimeters): $a_{\text {inp }}=8.636, b_{\text {inp }}=4.318$, $a_{s q}=8.636, l_{r w}=8.70, t=1.0, w_{1}=7.10, l_{1}=3.45, w_{2}=$ $5.90, l_{2}=3.84, w_{3}=4.61, l_{3}=3.73, w_{4}=2.58, l_{4}=1.63$, $a_{\text {out }}=8.636$.
(a)-(b) meet the specifications over two bands. The designed circular-output SP has the characteristics close to the specified ones as well (see Fig. 5). This follows partly from a fact that the bands are widely separated and the second resonance is sufficiently far from the beginning of high-frequency band in contrast to the SP of Fig. 3.

## IV. THE ANALYSIS OF POLARIZER WITH CORRUGATED HORN

Though the above-discussed higher-order modes are low in power, the following question has to be studied specially: how do they influence the resulting axial ratio value in the SP output? As an example, let us consider the SP illustrated by Fig. 3. For this SP, one $\left(\mathrm{TM}_{01}\right)$ or


Fig. 5. Performance of the designed four-step WR-34 septum polarizer with a circular output waveguide. Dimensions (in millimeters): $a_{\text {inp }}=8.636, b_{\text {inp }}=4.318$, $a_{s q}=8.40, l_{r w}=9.27, t=1.0, w_{1}=6.88, l_{1}=3.52, w_{2}=$ $5.63, l_{2}=3.65, w_{3}=4.34, l_{3}=3.40, w_{4}=1.95, l_{4}=1.43$, $l_{s q}=1.07, a_{t r}=8.6, l_{t r}=2.57, D=10.0$.
three $\left(\mathrm{TM}_{01}, \mathrm{TE}_{21}^{(c)}\right.$, and $\left.\mathrm{TE}_{21}^{(s)}\right)$ higher-order modes can propagate in the circular output over the 11.3 GHz 11.94 GHz or $11.94 \mathrm{GHz}-12.4 \mathrm{GHz}$ subbands of the high-frequency band, respectively (see Fig. 3 (b)).

We will further emphasize the second subband because of the greater number of higher-order propagating modes there. One can see that within this subband the $\mathrm{TM}_{01}$ and $\mathrm{TE}_{21}^{(s)}$ modes are excited with comparable amplitudes that are essentially higher than the $\mathrm{TE}_{21}^{(c)}$ mode one. However, these modes have the field component distributions and propagation constants differing from each other and from those of the dominant $T E_{11}^{(s)}$ and $T E_{11}^{(c)}$ modes. This will leads to a different axial ratio of the polarization ellipse of the


Fig. 6. The radiation (a) and polarization (b) patterns of the cascade consisting of the septum polarizer and corrugated conical horn.
entire field, i.e. including all the propagating higherorder modes, calculated at an arbitrary point of the SP circular output cross-section and at different distances from the output beginning. The axial ratio at any point of the output waveguide axis has to be equal to that calculated from the dominant modes amplitudes because the electrical field components of the considered higher-order modes are zero at the axis.

Due to the aforementioned entire field properties, it is difficult to obtain a certain answer to the above-put question. Such a conclusion is valid for the first subband with one high-order $\mathrm{TM}_{01}$ mode as well. More objective information can be obtained from the analysis of far field radiation and polarization patterns for a cascade of the septum polarizer and corrugated horn. A dual-band corrugated conical horn has been previously designed for two frequency bands identical to those for the polarizer of Fig. 3 using a numerical procedure reported in [18]. The horn is characterized by the following parameters: input waveguide diameter
$D_{\text {inp }}=24.4 \mathrm{~mm}$, aperture diameter $D_{\text {horn }}=132 \mathrm{~mm}$, semi-flare angle $\alpha=15^{\circ}$, throat circular waveguide section of the diameter $D_{\text {inp }}$ and the length $l$. Eight modes are taken into account in this section. It should be noted that the value of $l$ influences essentially the radiated field characteristics.

The radiation and polarization patterns of the polarizerhorn cascade with $l=8.7 \mathrm{~mm}$ are shown in Fig. 6(a) and 6(b), respectively. They are computed at $f=12.4$ GHz for several $\varphi$-planes within the 3 -dB $\theta$-angle sector. At the considered frequency, the polarizer has the worst axial ratio value $A X=-0.94 \mathrm{~dB}$ calculated from the dominant modes amplitudes and comparable amplitudes of the propagating $\mathrm{TM}_{01}$ and $\mathrm{TE}_{21}^{(s)}$ higherorder modes (see Fig. 3). Just these modes result in a deviation of the radiation pattern maximum (up to $\Delta \theta=0.8^{\circ}$ at $\varphi=90^{\circ}$ ) from the axial direction $\theta=0$ (Fig. 6(a)) and in an asymmetry of the polarization patterns in different $\varphi$-planes (Fig. 6 (b)). As expected, the axial ratio curves intersect at the point $\theta=0$ for all the $\varphi$-planes on the level equal to that for the polarizer (compare Fig. 3 (b) at $f=12.4 \mathrm{GHz}$ and Fig. 6 (b) at $\theta=0$ ). This is because the fields of higher-order modes do not make a contribution to the polarization patterns in the axial direction $\theta=0$. The symmetry condition of type $F(\varphi, \pm \theta)=F\left(180^{\circ}+\varphi, \mp \theta\right)$ is carried out for the relative power and axial ratio dependences on $\theta$ (for example, see curves for $\varphi=0$ and $\varphi=180^{\circ}$ in Fig. 6(a) and 6(b)).
One would expect that the similar features are valid for the radiation and polarization patterns of SPs with a square output as well.

## V. CONCLUSION

The results presented in this paper show the capabilities of the SP-CAD program as a tool for efficient and accurate design of compact waveguide septum polarizers terminated with square or circular output waveguides and operating in two distant frequency bands. The SP-CAD produces results that are in good agreement with those calculated with an extensively validated electromagnetic solver. The nature of the resonance effects limiting the bandwidths of different polarizer configurations is discussed. The influence of propagating higher-order modes on characteristics of the far field radiated from a corrugated conical horn terminating a polarizer is analyzed. It is found that
higher-order modes produce a deviation of the radiation pattern maximum and an asymmetry of the polarization pattern relative to the axial direction.

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# The Influence of Data Density on the Consistency of Performance of the Feature Selective Validation (FSV) Technique 

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#### Abstract

The human visual system has an immense capacity for compensating for poor or incomplete data. Psycho-visual coding schemes make use of the brain's ability to extrapolate and interpolate independently of conscious awareness to reduce data (bit) rates but maintain the same level of 'information' within a video signal. However, when attempting to produce a simple method for comparing data-sets, primarily for validation of computational electromagnetics, could give rise to a problem. Namely that someone undertaking the visual inspection of (e.g.) modeled data against experimental data will see the same picture whether sampled at $\mathrm{N}, 100 \mathrm{~N}$ or 0.01 N data points whereas the software undertaking the comparison would process three very different data sets. The Feature Selective Validation (FSV) method was developed to attempt to mimic the group response of a number of experts undertaking the visual comparison. Hence, the quality of performance of the FSV method should not be severely affected by the number of data points if this assertion is to hold, despite the obvious potential for variation. This paper investigates the FSV performance as a function of data density and shows that the accuracy of its performance remains largely unimpeded by variations in the precision of the data supplied.


## I. INTRODUCTION

In order for this paper to investigate the effect of data density on the FSV method, it needs to consider two issues. The first is to review the FSV method, clarifying what data is being used (including what this paper considers as data density), how it is being used, why and how the data density impacts on the underlying equations. The second factor is how does a normal graphical representation of data differ from how the data is presented to the FSV method. This section will overview the FSV method and the issues surrounding graphical representation and then lead on to a more detailed review of the FSV heuristics and then to tests to ascertain the quality of performance of the FSV method.

## I.1. FSV OVERVIEW

A typical scenario for the validation of computational electromagnetics involves the modeling of a system that can be directly measured. The resulting pair of data sets are then presented graphically to those involved in this exercise who will ascribe a quality level to the comparison: such as 'good'. Closer inspection may add a qualifier to this descriptor: such as 'good, but...'. The Feature Selective Validation (FSV) method was conceived as a technique to support this exercise, providing a numerical value to the quality of a comparison constructed from components analogous to the general approach used by humans. Namely, comparing the trends of the two data sets and comparing the individual features, perhaps resonant features, and combining these to give an overall confidence in the goodness of fit. The reason for the development of FSV over using existing methods, such as correlation, was that the existing methods do not offer sufficient discrimination or potential for feedback to the users.

The basis for the FSV method is to low pass and high pass filter the original two data sets, take differences of the low pass data to give the Amplitude Difference Measure (ADM) and take a mix of differences of derivatives of the low and high pass data to give the Feature Difference Measure (FDM). These represent the trend / envelope difference and the resonant-type feature difference discussed earlier. These are then treated as independent components and a Global Difference Measure (GDM) obtained from the ADM and FDM. More detail is given in section 2 and in references [1-3].

## I.2. REPRESENTATION ISSUES

When a set of measurements is taken and represented graphically, it is usual to present the data with lines joining the points, as in Figure 1.

Figure 2 shows the same data but represented more in the manner that a computer would 'see' it.


Figure 1. Normal presentation of data using lines.


Figure 2. "Non-interpolated" presentation of data.
While we look at continuous lines on a graph, the computer 'sees' points (more correctly a table of numbers). The presence of lines gives a sense of certainty to the locations in both ordinate and abscissa, however, representation as points shows the possibility of uncertainty in the location. While this is a trivial example, it can be seen that having fewer points on a graph increases the uncertainty, but presents less noise in the form of high frequency components. However, more data points increases the precision of the data but increases the noise present. Effectively, manipulating the number of data points may have virtually no effect on the visual representation of the data but will have a substantial effect on the data presented for analysis; for example halving the number of data points reduces the amount of data available for analysis by 3 dB but results in virtually no difference in the visual effect.

## I.3. PURPOSE OF STUDY

Bringing together the two themes discussed so far we can see that the use of a tool like the FSV method for the validation of CEM should, as far as possible, predict the response of a large group of users. By doing this, a
level of confidence can be attributed to the quality of the comparison. The response of the large group of users will be done 'by-eye' based on lines on a graph, suggesting continuous functions, further implicitly suggesting a level of precision that may not be realistically expected from the data. However, the response of the computer program will be obtained by following a clear and predetermined algorithm based on discrete points, i.e. non-continuous functions with the implication of uncertainty between these points.

Hence, the purpose of, and the research question for, this study is to see if varying the number of points presented to the FSV method will leave the output relatively unchanged, and certainly in line with user opinion. From this, it may be possible to issue some guidelines recommending good practice in the use of the FSV method so as to ensure a high level of consistency between applications.

There is the further issue that should be considered regarding the fact that as the number of data points used to represent the systems being compared is increased or decreased, the information content is either increased or decreased, the precision in the data (i.e. the tolerance of each point along the $x$ axis) varies and the noise content (or aliasing effect due to sampling effects) varies. Hence, while comparing data with, for example, 400 points in one instance and 100 points in another instance, may look identical, they are clearly separate sets of results. Issues surrounding data density and the veracity of the conclusions to be drawn from this are common in other walks of engineering (e.g. [4, 5]).

The next section will review the mathematics behind the FSV method and this will be followed by the tests to address the research question set out above.

## II. THE FSV METHOD

The FSV method was outlined above. This section reviews the governing equations and the methods used to represent the FSV output to users.

## II.1. FSV EQUATIONS

The governing equations are as follows. Note $x$ is the independent variable, $\mathrm{Lo}_{i}$ and $\mathrm{Hi}_{i}$ are the low pass and high pass filtered versions of $i^{\text {th }}$ data set $i=1,2$ (the subscript indicating the data set) and the single and double primes show the first and second derivatives with respect to $x$ obtained using a central difference scheme,

$$
\begin{align*}
& A D M(x)=\left|\frac{\left(\left|L o_{1}(x)\right|-\left|L o_{2}(x)\right|\right)}{\left.\frac{1}{N} \sum_{i=1}^{N}\left(\left|L o_{1}(i)\right|+\left|L o_{2}(i)\right|\right) \right\rvert\,}\right|  \tag{1}\\
& F D M(x)=2\left(\left|F D M_{1}(x)+F D M_{2}(x)+F D M_{3}(x)\right|\right)  \tag{2}\\
& F D M_{1}(x)=\frac{\left|L o_{1}{ }^{\prime}(x)\right|-\left|L o_{2}{ }^{\prime}(x)\right|}{\frac{2}{N} \sum_{i=1}^{N}\left(\left|L o_{1}{ }^{\prime}(i)\right|+\left|L o_{2}{ }^{\prime}(i)\right|\right)}  \tag{3}\\
& F D M_{2}(x)=\frac{\left|H i_{1}{ }^{\prime}(x)\right|-\left|H i_{2}{ }^{\prime}(x)\right|}{\frac{6}{N} \sum_{i=1}^{N}\left(\left|H i_{1}{ }^{\prime}(i)\right|+\mid H i_{2}{ }^{\prime}((i) \mid)\right.}  \tag{4}\\
& F D M_{3}(x)=\frac{\left|H i_{1} "(x)\right|-\left|H i_{2}{ }^{\prime \prime}(x)\right|}{\frac{7.2}{N} \sum_{i=1}^{N}\left(\left|H i_{1}{ }^{\prime \prime}(i)\right|+\left|H i_{2}{ }^{\prime \prime}(i)\right|\right)}  \tag{5}\\
& G D M(x)=\sqrt{A D M(x)^{2}+F D M(x)^{2}} . \tag{6}
\end{align*}
$$

In the summary, single value representations of the ADM, FDM, and GDM are obtained by taking the mean value over the range of $x$ of interest.

Given that a central difference scheme has been used for the derivatives, it is assumed that the data points are evenly spaced. So, from Table I it can be seen that

$$
\begin{equation*}
y^{\prime}(x 4)=(y(x 5)-y(x 3)) /(x 5-x 3) . \tag{7}
\end{equation*}
$$

Table I. Representation of trial data.

| Point number | Data value |
| :--- | :--- |
| 1 | $y\left(x_{1}\right)$ |
| 2 | $y\left(x_{2}\right)$ |
| 3 | $y\left(x_{3}\right)$ |
| 4 | $y\left(x_{4}\right)$ |
| 5 | $y\left(x_{5}\right)$ |
| 6 | $y\left(x_{6}\right)$ |

However, in an undersampled version of this, as indicated in Table II

$$
\begin{equation*}
y^{\prime}(x 4)=(y(x 6)-y(x 5)) /(x 6-x 5) . \tag{8}
\end{equation*}
$$

Table II. Under sampled data of Table I.

| Point number | Data value |
| :--- | :--- |
| 3 | $Y\left(x_{2}\right)$ |
| 4 | $y\left(x_{4}\right)$ |
| 5 | $Y\left(x_{6}\right)$ |

There are two practical implications for using this data in FSV. The first is that where there is a low rate of change in the data, then $y\left(x_{5}\right)-y\left(x_{3}\right) \approx y\left(x_{6}\right)-y\left(x_{5}\right)$ and a high rate of change may render this approximation incorrect. The second point is that in Table II the separation of data points is $2 \Delta x$, where $\Delta x$ is the separation in Table I. However, in the FSV equations employing derivatives, the derivatives appear in both the numerator and denominator of the equations, so the $\Delta x$ effect will cancel. Of course, this is only true if the $\Delta x$ used for derivatives of dataset 1 is equal to the $\Delta x$ used for derivatives of dataset 2 . This assumption has been made in this analysis.

This leaves the issue of whether $\Delta y_{\mathrm{a}}$ is sufficiently close to $\Delta y_{\mathrm{b}}$ where a and b represent two different sampling rates. This will be implicitly investigated in the next section.

In order to help interpret the results in the next section, methods used to represent the FSV data to users will be reviewed.

## II.2. FSV REPRESENTATION

The basic representation of the FSV output can be either simply numeric (i.e. single figure values for the ADM, FDM and GDM) or point-by-point values ( $\{\mathrm{A}$, $\mathrm{F}, \mathrm{G}\} \mathrm{DM}(x)$ as in the previous equations). However, one of the design requirements for FSV was to provide a range of diagnostic information [2]. Bearing in mind the aim for the FSV method to mirror the opinions of a group of engineers, it has been found that the proportions of each of the measures that falls into the definitions of the natural language descriptors, given in Table III, provides a useful histogram which is suggestive of the proportions of a large group who, when assessing the original comparisons would categorize them according to the categories [3]. These are called the confidence histograms for each of the measures.

Table III. Natural language descriptors for FSV \{A, F, G\}DM.

| FSV (quantitative) value | Natural descriptor | language |
| :---: | :---: | :---: |
| Value $\leq 0.1$ | Excellent |  |
| $0.1 \leq$ Value $<0.2$ | Very good |  |
| $0.2 \leq$ Value $<0.4$ | Good |  |
| $0.4 \leq$ Value $<0.8$ | Fair |  |
| $0.8 \leq$ Value $<1.6$ | Poor |  |
| $1.6 \leq$ Value | Very poor |  |

Given that the confidence histograms aim to provide a synthetic group response, a wide spread of similar
height categories would lead to the interpretation that there can be a low confidence in attributing a single epithet to a comparison. For example, if the confidence histogram showed an approximately even distribution between Good, Fair, and Poor, it would be inappropriate to describe the comparison as Fair; it would be better to describe it as Good - Poor. In order to capture this in a more algorithmic manner, the 'spread' of the confidence histograms has been introduced [6]. The Spread is the number of categories that contains $85 \%$ of the data points (taking the difference between the most and least favourable category). So, for example, if in the previous example, $30 \%$ of the points fell into each of the Good, Fair, and Poor categories, then the Spread would be 3. If, on the other hand, approximately half fell into the excellent category and nearly half into the Very Poor category, then the spread would be 6 . Effectively, the Spread is a measure of the variance of the histogram data.

In order to balance the Spread, where a Spread of 2 could equally result from a combination of ExcellentVery Good as it could from Poor - Very Poor, a Grade measure has also been introduced alongside the Spread. Whereas the Spread could be thought of as a variance measure, the Grade is similar to an upper action line in process control. The Grade is the number of categories, starting with Excellent that need to be included for $85 \%$ of the data points in the particular measure to be counted. Thus, in the Good-Fair-Poor illustration the Grade would be 5; in the Excellent-Very Good illustration, the Grade would be 2 but the Poor-Very Poor illustration would have a Grade of 6 . So together, the Grade-Spread gives a simple indication of the quality and reliability of the comparison.

The following analysis uses Grade-Spread in addition to the summary values to quantify the differences that varying the data density has on the comparison results. The results were obtained using a stand-alone FSV application [7, 8].

## III. TESTS

In order to assess the performance of FSV when faced with varying data density, tests with three different data types have been performed. These are (1) EMC modelling of a via performance (2) very high feature density performance and (3) sinusoid representations. In doing this (1) is representative of real data that will commonly be presented to FSV, (2) is representative of data at one extreme of complexity and (3) is representative of data at the other extreme of complexity.

## III.1. VIA PERFORMANCE

The system was modeled with 5768 points per data set, this is shown in Figure 3, and the data was then down sampled to a minimum of 177 points: Figure 4.


Figure 3. Via models: 5768 samples.


Figure 4. Downsampled data of Figure 3: 177 samples.
Table IV. FSV summary values for various data densities between the representations of Figures 3 and 4.

|  | ADM | FDM | GDM |
| :--- | :--- | :--- | :--- |
| No of Data <br> points |  |  |  |
| 5768 | 0.165 | 0.604 | 0.665 |
| 2839 | 0.152 | 0.536 | 0.587 |
| 1419 | 0.139 | 0.395 | 0.448 |
| 709 | 0.132 | 0.321 | 0.374 |
| 384 | 0.132 | 0.277 | 0.332 |
| 177 | 0.140 | 0.274 | 0.334 |

Initial visual observations of this data suggest that the downsampling, as would be expected, has reduced the level of 'high-Q' features resulting in an overall visual improvement in the Feature (i.e. the 'high-Q' aspect) component of the original data, leaving the Amplitude (i.e. the envelope / trend) information relatively unaffected. The effect of the changes can be seen in Table IV which lists the ADM, FDM and GDM components for various data densities.

From a natural language descriptor equivalent, the ADM is unchanged at Very Good, whereas the GDM has gone from Fair to Good, reflecting a likely visual analysis.

However, in order to address possible researcher bias, the Grade-Spread information, as discussed in Section 2 was noted. For all the data densities, the ADM Grade remained at 3 and the Spread at 3. On the other hand, the Grade for the FDM and GDM was 5 for 5768 points -709 points inclusive and was 4 for the remaining two comparisons. The Spread was mostly 4 for the FDM and GDM (it did nudge into 5 for the FDM and GDM of 2839 samples and for the FDM of 1419 samples). However, together this data suggests that the FSV routine is only particularly sensitive to the number of data points used as far as the eye is sensitive to the filtering and smoothing effects of the reduction in the number data points used to present the graphs.

## III.2. HIGH FEATURE DENSITY

In this test, very highly structured and noisy data was downsampled. Figure 5 gives the original data and Figures 6 and 7 give undersampled versions of this data.


Figure 5. Original data: 3000 samples.
From these three figures, it is clear that the general structure of the data becomes visibly clear once the
original data has been downsampled to $50 \%$. Once the structure has become visibly clearer, it is not immediately obvious what effect the reduction in data density will have on the quality of comparison. Table V gives the Difference Measures for various data densities.


Figure 6. Downsampled data of Figure 5: 1500 samples.


Figure 7. Downsampled data of Figure 5: 97 samples.

Table V. FSV summary values for various data densities between the representations of Figures 5 and 7.

|  | ADM | FDM | GDM |
| :--- | :--- | :--- | :--- |
| No of Data <br> points |  |  |  |
| 3000 | 0.354 | 0.609 | 0.772 |
| 1500 | 0.547 | 0.681 | 0.977 |
| 750 | 0.550 | 0.709 | 0.995 |
| 375 | 0.521 | 0.658 | 0.928 |
| 187 | 0.558 | 0.649 | 0.937 |
| 93 | 0.584 | 0.727 | 1.027 |

It is interesting to note that as the graphs become visibly more structured, i.e. moving from Figure 5 to Figure 6, the FSV analysis becomes reasonably constant (below one decimal place). This is interesting because the similarities between Figures 6 and 7 are well defined and there is a high probability that they would be noted as such by most users. However, the similarity between Figures 5 and 6 are not so easy to see.

This observation is also reflected in the Grade-Spread. Excluding the 3000 point comparison, the grade is constant at 5 for the ADM, GDM and FDM for all comparisons except the GDM for 750 and 93 where it tips over to 6 . Never-the-less a consistent performance.

For the Spread, the FDM is constant at 5 and the ADM and GDM both go from 4 ( 1500 sample) to 3 ( 93 sample). These results reflect the effect of reducing the data density on the visual interpretation, increasing the probability for consistency between users.

So far, the tests have only shown that the FSV method reflects the likely interpretation of the results by expert users because this data is highly structured and, therefore, relatively easy to identify visual changes to the structure of the data. The more demanding task, in these circumstances, is to take a simple structure, which will barely change visually and analyze this with FSV.

## III.3. SINUSOIDS

In this final test, two sinusoids were compared after differences were introduced between them. These differences were changing the amplitude so one was $10 \%$ of the amplitude of the other and secondly, one was $90 \%$ of the amplitude of the other; the final test was to introduce a phase shift between the two curves.

## III.3.1. 10\% Relative Amplitude

The two curves, one at peak amplitude of 10 V and the other with peak amplitude of 1 V were compared over one period with a range of data points from 50 to 800 . The comparison of these curves is illustrated in Figure 8. The summary values for the ADM, FDM and GDM are listed in Table VI.

Clearly, there is a consistency in all the figures. The slight variations in the GDM arise through the point-bypoint nature of the summation and are illustrated in the confidence histogram in Figure 9.


Figure 8. Two sinusoids of very different amplitude.

Table VI. FSV summary values for various data densities for a 10 V and 1 V sinusoid.

|  | ADM | FDM | GDM |
| :--- | :---: | :---: | :---: |
| No of Data <br> points |  |  |  |
| 800 | 1.091 | 1.318 | 1.803 |
| 400 | 1.091 | 1.318 | 1.802 |
| 200 | 1.091 | 1.318 | 1.811 |
| 100 | 1.091 | 1.318 | 1.823 |
| 50 | 1.318 | 1.831 |  |



Figure 9. GDM confidence histogram for two sine waves of 10 V and 1 V amplitude.

## III.3.2. 90\% Relative Amplitude

The same test as in section 3.3.1 was performed, but this time 10 V and 9 V amplitude sine waves were compared, as illustrated in Figure 10. Table VII lists the summary values.

Again, there is practically very little difference in the results, as illustrated with the GDM confidence histogram in Figure 11


Figure 10. Two sinusoids of similar amplitude.
Table VII. FSV summary values for various data densities for a 10 V and 9 V sinusoid.

|  | ADM | FDM | GDM |
| :--- | :---: | :---: | :---: |
| No of Data <br> points |  |  |  |
| 800 | 0.071 | 0.127 | 0.156 |
| 400 | 0.07 | 0.102 | 0.132 |
| 200 | 0.07 | 0.086 | 0.118 |
| 100 | 0.07 | 0.085 | 0.117 |
| 50 | 0.07 | 0.085 | 0.118 |

The greatest difference that can be seen from this figure is that there has been some shifting between Excellent and Very Good for the 800 point data. This is probably because of the proportion of the GDM at approximately 0.1 , i.e. the boundary between the two categories.


Figure 11. GDM Confidence histogram for two sine waves of 10 V and 9 V amplitude.

## III.3.3. Phase Difference

The last of the sinusoidal tests involved taking two similar sinusoids and adding in a one radian phase shift. To add additional difficulty in the tests, the comparison was made over two cycles of the original data with data densities ranging from 10000 points to 20 points. In the case of 20 points, i.e. 10 data points per cycle, the curves clearly depart from smooth sinusoids. The 20 data point curve is shown in Figure 12 and the 100 data point curve, for comparison, is shown in Figure 13.


Figure 12. Two poorly defined sinusoids, each with 10 samples per cycle.


Figure 13. Two well defined sinusoids.
The comparison of the Difference Measure results is given in Table VIII.

Table VIII. FSV summary values for various data densities for two offset sinusoids.

|  | ADM | FDM | GDM |
| :--- | :--- | :--- | :--- |
| No of Data <br> points |  |  |  |
| 10000 | 0.673 | 0.894 | 1.074 |
| 1000 | 0.674 | 0.689 | 1.063 |
| 100 | 0.674 | 0.722 | 1.094 |
| 20 | 0.712 | 0.844 | 1.206 |

Figure 14 shows the effect of changing the data densities on the GDM.


Figure 14. GDM for two offset sinusoids as a function of data density.

Figure 14 shows how consistently the FSV method assesses the comparison of these two data sets.

## IV. CONCULSIONS

This paper has considered the effects of varying data density on the results of FSV analysis of varying data sets.

The FSV method is a promising approach to the formal and quantifiable comparison of data for tasks such as computational electromagnetic (CEM) validation, experimental repeatability and assessment of the amount of change when models are manipulated. However, as part of the validation of FSV itself, its robustness to changes in input data needs to be assessed. This paper has done this with three different, typical data sets, ranging from the structurally trivial (sinusoids) to the structurally highly complex.

The main conclusion that that can be drawn from the results is that FSV is robust in that it does not change the output values appreciably when the input data density changes, providing the data remains visually unchanged. This was clearly seen with the sinusoidal data where the overall levels of agreement did not change substantially once the data density was past the point where it appeared smooth and continuous.

The important general recommendation from this work is that there should be consistency in the number of points used to represent the data and that number should be chosen because of the way in which the data is represented. That is, it is important to choose the correct number of points to represent the data to be compared so that the visual representation is precisely that which needs to be compared. It was clear with the first two examples in this paper that the effective data to be compared could be changed considerably depending on whether more or fewer points were displayed. Clearly, this factor has more of an impact on the assessment of the quality of a comparison than anything else. Perhaps a standard question during validation should be "why has the data been displayed with this level of data density?"

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# To-Average or Not-to-Average in FDTD Modeling of Dielectric Interfaces 

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#### Abstract

Accurate Finite Difference Time Domain (FDTD) modeling of localized electromagnetic sources such as cellular telephones near the human body requires very precise modeling of the location of these devices. This paper discusses the effective location of near field sources when dielectric interface cells are made up of either averaged or unaveraged dielectric properties. It is shown that either method can accurately define the proximity between source and dielectric, but that the effective location differs by half an FDTD cell in the two methods.


## I. INTRODUCTION

As Finite Difference Time Domain (FDTD) modeling of the human body and other complex dielectric systems becomes main stream [1], [2], many researchers have passionately defined a preference for either averaging or not averaging dielectric materials at boundaries. This paper discusses this issue from the point of view of very near field modeling, where the precise location of a source such as an antenna near the human head is critical to the accuracy of the results. This modeling issue has potentially significant implications in cellular telephone evaluation and the ongoing debate about if power deposition from cell phones is or is not higher in children than in adults [3]. Little or no difference between power absorbed from cellular telephones in adult and child head models is seen if the antenna is modeled an equal distance from the outer surface of both head models [4]. However, if the phone is modeled as being physically closer to the child's head due to the thinner ear of the child, significantly higher power is observed in the child than adult model [5]. This is not surprising, since the fields decrease rapidly (as $1 /$ distance $^{3}$ ) very near the antenna, and small changes in the proximity of the phone can result in large changes in power deposition. In addition to the effect of considering or not considering the size of the ear, averaging or not averaging the dielectric properties at the outer surface of the head effectively moves the boundary half a cell, with potentially similar results. When evaluating near field effects such as the power deposition of a cell phone in a human head, it is critical to model the proximity of the source accurately. This requires an understanding of how the choice of averaging or not averaging FDTD dielectric boundaries
affects the effective proximity of the source, which is the topic of this paper.

Several different methods have been used for averaging electrical properties at boundaries of dissimilar media. When a layer of dielectric is thinner than one FDTD cell, averaging is one method of accounting for the thin layer [6]-[10]. Averaging has also been used for boundaries in an effort to reduce unwanted effects from discontinuity of the charge at the boundary [11]-[14]. Interpolation of the fields rather than the model has been proposed as an alternative to averaging [15]. Averaging at the air-dielectric interface of microstrip circuits and antennas has been used routinely [16], [17]. In [18] it was found that averaging was not needed for buried layered structures with no conductor, but that it was helpful when a conductor rested on the air-dielectric interface. Three different types of averaging are used (arithmetic, geometric, and harmonic), and their relative accuracy has been shown to be dependent on the cell size [9], [13]. All of these papers were focused on the effect of the dielectric discontinuity on the model of the dielectric object where the distance between the boundary and source is large enough that a variation of half of an FDTD cell would be negligible. When models of antennas near a dielectric object are considered, the ability of averaging to improve the model is found to be minor, especially when compared to variation in the source location of the antenna [19]. It was also commented in [20] on the effect that averaging can have on displacing the boundary by half a cell.

This paper discusses the effect of averaging on the effective location of a source or boundary and demonstrates that either an averaged and non-averaged dielectric boundary can be used to model the source and boundary in their proper locations. This paper demonstrates the effect of averaging or not averaging the dielectric boundary by considering the phase of the reflection coefficient in front of the boundary. This derivation could have been done analytically as in [2], Section 3.6.8, and the same conclusion could have been drawn.

## II. NEAR-FIELD FDTD MODELING

The analysis of modeling the proximity of a source to a dielectric object will be limited in this paper to one dimension in order to precisely demonstrate the effect of
averaging, although the conclusion is easily generalized to 2 D and 3D simulations. The one-dimensional TE FDTD grid is shown in Figure 1 for unaveraged cases (a) and (c) and averaged case (b). The locations of two different materials, 1 and 2, in the model are indicated as well as the average of materials 1 and 2 in the center of (b). The location of the physical boundary of each model is indicated by the arrow, as will be shown in the results section.


Fig. 1. FDTD model of a dielectric interface; (a) and (c) are unaveraged models, (b) is an averaged model. The location of the physical boundary that each model represents is shown with the arrow. The location of the fields in the 1D FDTD cell are shown as well. The two materials modeled were air $\left(\varepsilon_{\mathrm{r} 1}=1\right)$ and "water" ( $\varepsilon_{\mathrm{r} 2}=$ 40.0). Both materials are lossless.

Maxwell's differential equations in the time domain are

$$
\begin{gather*}
\nabla \times \overline{\mathrm{E}}=-\mu \frac{\partial \overline{\mathrm{H}}}{\partial \mathrm{t}}  \tag{1}\\
\nabla \times \overline{\mathrm{H}}=\sigma \overline{\mathrm{E}}+\varepsilon \frac{\partial \overline{\mathrm{E}}}{\partial \mathrm{t}} . \tag{2}
\end{gather*}
$$

They are simplified to two 1D scalar Maxwell's curl equations:

$$
\begin{align*}
& \frac{\partial \mathrm{E}_{\mathrm{y}}}{\partial \mathrm{t}}=\frac{-1}{\varepsilon}\left(\frac{\partial \mathrm{H}_{\mathrm{Z}}}{\partial \mathrm{x}}+\sigma \mathrm{E}_{\mathrm{y}}\right)  \tag{3}\\
& \frac{\partial \mathrm{H}_{\mathrm{Z}}}{\partial \mathrm{t}}=\frac{-1}{\mu}\left(\frac{\partial \mathrm{E}_{\mathrm{y}}}{\partial \mathrm{x}}\right) \tag{4}
\end{align*}
$$

where $\varepsilon$ in (3) is defined either as averaged (Figure 1b) or unaveraged (Figures 1a and 1c) for cells at the
boundary. For simplicity, $\sigma=0$ in these test cases. In the non-averaged cases, (a) or (c), $\varepsilon$ is $\varepsilon_{1}$ or $\varepsilon_{2}$, depending on the cell. In the averaged case (b), the value of $\varepsilon$ is $\varepsilon_{1}$ or $\varepsilon_{2}$ on either side of the boundary and the average value of the two materials $\left(\varepsilon_{2}+\varepsilon_{1}\right) / 2$ for the boundary cell. It should be noted that this averaged model is the same as modeling a 1-cell thick layer of ( $\varepsilon_{2}$ $\left.+\varepsilon_{1}\right) / 2$ dielectric material sandwiched between two different dielectrics ( $\varepsilon_{1}$ and $\varepsilon_{2}$ ), the boundaries of which would be where the arrows are shown in Figures 1a and c. This is not the most accurate way of modeling a thin layer, however, and is not recommended for applications where the thin layer is expected to have a significant effect on the performance of the system.

To find the phase of the reflection coefficient, the discrete Fourier transform (DFT) of the electric field in every cell was calculated at 1 MHz . This provided the total electric field, $\mathrm{E}^{\mathrm{t}}$. The DFT at each test cell was calculated for each of the three boundary arrangements, and it was also calculated with no dielectric boundary (i.e., with air in all of the cells). The latter gave the DFT of the incident wave alone ( $\mathrm{E}^{\mathrm{i}}$ ), including any effective errors from numerical dispersion. The reflection coefficient at each point can be calculated using

$$
\begin{equation*}
\Gamma_{\ell}=\frac{E^{t}-E^{i}}{E^{i}} \tag{5}
\end{equation*}
$$

where $E^{t}$ and $E^{i}$ are the DFTs of the total and incident wave, respectively. Analytical values for the test points were calculated using

$$
\Gamma_{\ell}=|\Gamma| \mathrm{e}^{\mathrm{j} 2 \beta \ell}
$$

where $\ell$ is the distance from the center cell (where the arrow is shown in Figure 1a) to the test point and

$$
\begin{equation*}
|\Gamma|=\frac{\sqrt{\varepsilon_{\mathrm{r} 1}}-\sqrt{\varepsilon_{\mathrm{r} 2}}}{\sqrt{\varepsilon_{\mathrm{r} 1}}+\sqrt{\varepsilon_{\mathrm{r} 2}}} \tag{6}
\end{equation*}
$$

## III. RESULTS

Figure 2 shows a comparison of the phase as a function of distance from the boundary (where the arrow is in Figure 1b) for all three models along with the analytical solution. The uppermost and lowermost lines were produced by the unaveraged models (a) and (c), respectively. The middle line was produced by (b). The analytical value for a boundary at the location of the arrow in Figure 1b is also give. Since the line for the averaged model is halfway between lines for the unaveraged model, we can see that averaging has the effect of shifting the boundary one half-cell away from the source. When the analytical value is calculated at
the location of the arrows shown in Figures 1a and 1c, the match with those curves is as good as that shown here for Figure 1b. The only difference between the averaged and unaveraged models is the phase of the reflection coefficient (which is indicative of the effective location of the dielectric boundary). All three models can properly model the location of the boundary and will give the same phase as the analytical solution if the location of the boundary is taken to be wherever their respective arrows indicate.


Fig. 2. Phase of reflection coefficient as a function of normalized distance from the (b) boundary (where arrow is seen in Figure 1b). Phase is shown for unaveraged (a and c) and averaged (b) FDTD models. The analytical solution is shown for a boundary where the arrow is shown in Figure 1b. When the analytical solution is calculated relative to the (a) or (c) boundaries (where the arrows are located in Figs. 1a and 1c), it lines up precisely with those lines, instead. This provides proof that both averaged and unaveraged models can be used to accurately model boundaries in FDTD, but that changing the averaging scheme changes the effective location of the boundary.

## IV. CONCLUSIONS

Near field FDTD simulations such as models of a cell phone near the human head depend significantly on the proximity of the antenna feedpoint to the head. Properly modeling this requires precise attention to the distance between the source and model, which varies by half a cell depending on if the electrical properties on the boundary are averaged or not. In this paper it was shown that both averaged and unaveraged models can be used with equal accuracy, as long as the model takes into account the location of the boundaries for each case, shown as arrows in Figure 1.

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