Semi-analytical Approach to Sensitivity Analysis of Lossy Inhomogeneous Structures

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Abstract – We propose an adjoint-variable technique for sensitivity analysis with structured-grid EM solvers, which can handle lossy inhomogeneous materials. In previous discrete adjoint-based approaches, the response derivatives with respect to shape parameters require the solution of a perturbed geometry, which has to be approximated. Here, we improve the algorithm by proposing a semi-analytical sensitivity formula where the system matrix derivatives consist of an analytical and a finite-difference term. It allows the use of the solution of the unperturbed structure with no approximation, which improves the accuracy. Applications are based on a frequency-domain solver based on the transmission line method.

Index Terms – Sensitivity analysis, adjoint variable method, lossy dielectrics, complex permittivity, and frequency-domain transmission line method.

I. INTRODUCTION

The adjoint variable method (AVM) is the most efficient method for sensitivity analysis, see, e.g., [1] - [6]. Discrete adjoint-based techniques (DAVM) have been proposed for implementation with high-frequency structured-grid time-domain [3] -[5] and frequency-domain [6]], [7] solvers. The response gradient is computed with two full-wave simulations – of the original problem and the adjoint problem – regardless of the number of parameters of interest.

All preceding discrete adjoint-based techniques have been developed for implementation exclusively with loss-free homogenous problems, where the parameters of interest are either shape parameters of perfectly conducting details [3] - [6] or dielectric details [5], [7]. All these techniques require either the solution of the perturbed original problems [6], [7] or the solution of the perturbed adjoint problems [3] - [6]. The solution to these perturbed problems is approximated using a simple mapping between the original and the perturbed field solutions [3], [4], [6]. This is necessary to avoid the K additional full-wave analyses, K being the number of parameters of interest.

Most practical problems involve lossy inhomogeneous mediums. Examples include detection of tumors in the human body, nondestructive testing and evaluation (NDT/NDE) [8], etc. The solution to these problems often employs optimization algorithms whose efficiency is significantly improved by the availability of the response gradient.

In this paper, we address the sensitivity analysis for problems involving lossy inhomogeneous materials. We first investigate the application of our preceding discrete adjoint-based technique [6], [7] to the sensitivity analysis of lossy inhomogeneous problems. Such an investigation is carried out for the first time. This technique requires the approximation of the solutions of the perturbed original problems. Next, we propose an improvement of this technique based on a semi-analytical sensitivity formula. The newly developed formula requires the solution of the original unperturbed problem only. Hence, it avoids any solution approximations. This approach allows us to obtain sensitivities with respect to shape parameters using the complex permittivity as well as its real or imaginary part as intermediate variables in the new sensitivity expression.

We start by giving a brief description of the frequency-domain transmission line method and its implementation with lossy and inhomogeneous materials in section II. In section III, we give a brief background of preceding adjoint-based techniques. Our proposed semi-analytical formula for sensitivity analysis is given in section IV. Verification results are presented in section V. Conclusions are drawn in section VI.

II. THE FREQUENCY-DOMAIN TLM METHOD WITH LOSSY DIELECTRICS

The frequency-domain transmission line method (FDTLM) [9] carries out a sequence of scattering and connection processes among its TLM symmetrical condensed node(s) (SCN) in a similar way as in the time-domain transmission line method (TDTLM) [10]. An equation is written for each link of the SCN relating its voltages to those from neighboring nodes. These voltages are delayed through propagation factors of the type $e^{-\gamma_l \delta/2}$ where δ is the node size along the lth link. The propagation constant γ_l is

$$\gamma_l = j\omega\sqrt{\mu\tilde{\varepsilon}} \tag{1}$$

where

$$\tilde{\varepsilon} = \varepsilon_o \varepsilon_r - j\sigma/\omega . \tag{2}$$

In equation (1), ω is the angular frequency; $\tilde{\varepsilon}$, ε_r, μ and σ are the complex permittivity, the relative permittivity, the permeability and the conductivity, respectively. Local variations in the material properties are modeled by modifying the propagation constants of the node links. With lossy materials, the voltage waves are multiplied by a complex exponential factor.

When the equations for all the links in all the nodes are put together, we arrive at

$$\boldsymbol{A} \cdot \boldsymbol{v} = \boldsymbol{V}_s . \tag{3}$$

Here, $\boldsymbol{A} \in \mathbb{C}^{12N_t \times 12N_t}$ is the system matrix, $\boldsymbol{v} \in \mathbb{C}^{12N_t \times 1}$ is the solution vector containing all incident voltages in the computational domain, and $\boldsymbol{V}_s \in \mathbb{C}^{12N_t \times 1}$ is the excitation. All quantities in equation (3) are complex with N_t being the total number of TLM nodes in the computational domain.

III. BACKGROUND

Our objective is to efficiently compute the sensitivity of a response function $f\left(\boldsymbol{x}, \boldsymbol{v}(\boldsymbol{x})\right)$ with respect to changes in the vector of parameters $\boldsymbol{x} = [x_1 \ \dots \ x_K]^T$, i.e., $\nabla_{\boldsymbol{x}} f$. Note that $\nabla_{\boldsymbol{x}}$ is defined as a row operator [1],

$$\nabla_{\boldsymbol{x}} f = \left[\frac{df}{dx_1} \ \cdots \ \frac{df}{dx_K} \right]. \tag{4}$$

The elements in $\boldsymbol{x}$ could be related to the material parameters of the structure and/or to its shape. We introduce in this section possible implementations of the AVM for sensitivity analysis with full-wave frequency-domain solvers.

A. *The Exact AVM*

With the exact AVM, the response derivative with respect to the kth parameter x_k is obtained as [2], [6]

$$\frac{df}{dx_k} = \frac{\partial f}{\partial x_k} + \boldsymbol{\lambda}^H \cdot \left[\frac{\partial \boldsymbol{V}_s}{\partial x_k} - \frac{\partial \boldsymbol{A}}{\partial x_k} \cdot \boldsymbol{v} \right], \quad k = 1, \ \dots, K. \tag{5}$$

In equation (5), $\partial/\partial x_k$ represents an explicit dependence on x_k; $\boldsymbol{v}$ is the solution of equation (3); and $\boldsymbol{\lambda}^H$ is the Hermitian of the adjoint variable vector $\boldsymbol{\lambda}$. It is the solution of the adjoint problem

$$\boldsymbol{A}^H \cdot \boldsymbol{\lambda} = [\nabla_{\boldsymbol{v}} f]^H . \tag{6}$$

Here, $\boldsymbol{A}^H$ is the Hermitian of the system matrix $\boldsymbol{A}$. $\nabla_{\boldsymbol{v}} f$ is the adjoint excitation, which is the gradient of the response f relative to the state variables in $\boldsymbol{v}$.

The sensitivity equation (5) is exact in the sense that all three required derivatives – including the system matrix derivative $\partial \boldsymbol{A}/\partial x_k$ – are exact, i.e., analytic. The accuracy of the computed sensitivities is affected by the accuracy of the solution vectors $\boldsymbol{\lambda}$ and $\boldsymbol{v}$, which depends on the accuracy of the numerical solver.

It is straightforward to use equation (5) for sensitivity estimates with respect to material parameters of the structure such as the constitutive parameters ε_r and σ. Its implementation with shape parameters, however, is very limited [7].

B. *The Discrete AVM*

With structured grid solvers $\partial \boldsymbol{A}/\partial x_k$ is rarely available analytically for shape parameters. The discrete AVM (DAVM) overcomes this limitation. It determines the derivative with respect to the kth parameter x_k as in [7],

$$\frac{df}{dx_k} \approx \frac{\partial f}{\partial x_k} + \boldsymbol{\lambda}^H \cdot \left[\frac{\Delta_k \boldsymbol{V}_s}{\Delta x_k} - \frac{\Delta_k \boldsymbol{A}}{\Delta x_k} \cdot \tilde{\boldsymbol{v}}_k \right], \quad k = 1, \ \dots, K. \tag{7}$$

Here, $\Delta_k \boldsymbol{A}$ is the difference matrix due to a stepwise perturbation of $\Delta x_k = \delta$ in x_k. The difference ratio $\Delta_k \boldsymbol{A}/\Delta x_k$ is, in general, not a good approximation of $\partial \boldsymbol{A}/\partial x_k$, and using it directly with equation (5) leads to unacceptable errors. Therefore, the solution of the kth perturbed problem $\tilde{\boldsymbol{v}}_k$ is required with such discrete perturbations. The perturbed solutions $\tilde{\boldsymbol{v}}_k$, $k = 1, \ \dots, K$, are not obtained by actually solving the problems, since this would require K additional system analyses. They are approximated through

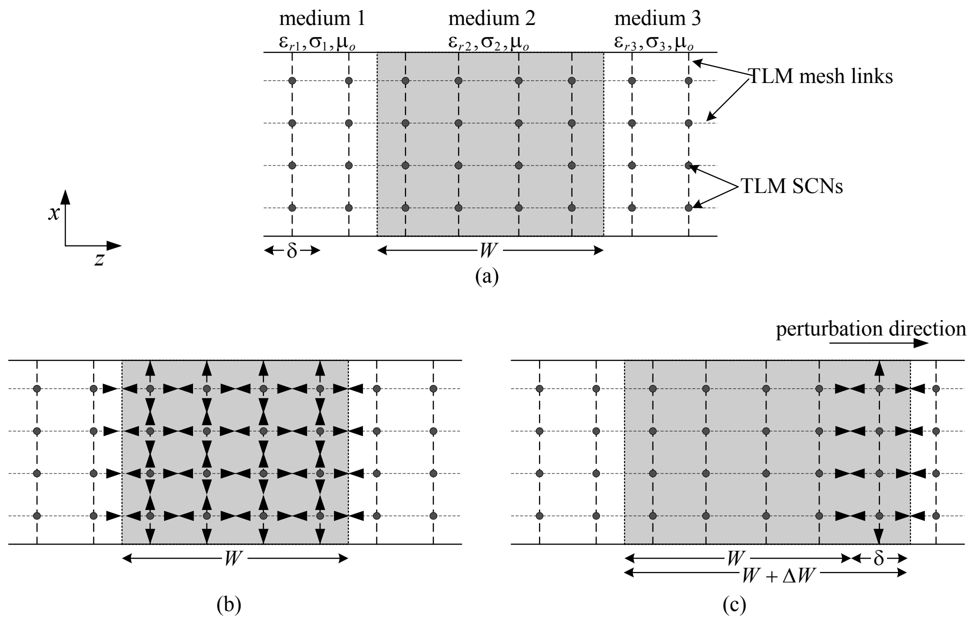

Fig. 1. Computing the analytical derivative of the system matrix $\partial \boldsymbol{A}/\partial \tilde{\varepsilon}$ for response sensitivity evaluation (2D view): (a) the unperturbed problem, (b) the nonzero elements of $\partial \boldsymbol{A}/\partial \tilde{\varepsilon}$ when computing $df/d\tilde{\varepsilon}$ with the exact AVM approach, and (c) the nonzero elements of $\partial \boldsymbol{A}/\partial \tilde{\varepsilon}_n$ when computing df/dW with the semi-analytical AVM approach.

mappings of the solution of the unperturbed problem (3). Hence, equation (7) requires K approximations of $\tilde{\boldsymbol{v}}_k$ ($k=1, \ldots, K$) [7].

The discrete sensitivity, equation (7), is advantageous over the exact, equation (5), because of its versatility. It can be used for sensitivities with respect to material-related parameters as well as shape parameters. Its implementation with lossy materials deserves special consideration and is addressed here for the first time.

IV. THE SEMI-ANALYTICAL SENSITIVITY FORMULA

An appealing feature of the TLM solver in the frequency domain is the direct analytical relation between the system matrix $\boldsymbol{A}$ and the constitutive parameters of the SCNs. This relation is identified through the propagation factor γ_l as well as the transmission and reflection coefficients at the boundaries between the different regions in the computational domain, see equations (1) to (3). This feature allows us to derive an alternative approach to equation (4) in the case of lossy dielectric discontinuities. The approach combines the exact AVM and the DAVM. It employs the analytical dependence of the system matrix on the material properties of the SCNs. No approximations are required as in equation (7) and, unlike equation (5), it is versatile as it can be implemented with shape and material parameters.

Let $\{1, \ldots, m\}$ be the set of node indices whose dielectric properties change as the shape parameter x_k changes. We rewrite equation (5) as

$$\frac{df}{dx_k} = \frac{\partial f}{\partial x_k} + \boldsymbol{\lambda}^H \cdot \left[\frac{\partial \boldsymbol{V}_s}{\partial x_k} - \sum_{n=1}^{m} \left(\frac{\partial \boldsymbol{A}}{\partial \tilde{\varepsilon}_n} \frac{\partial \tilde{\varepsilon}_n}{\partial x_k} \right) \cdot \boldsymbol{v} \right], \quad k=1, \ldots, K. \tag{8}$$

As before, the adjoint variable $\boldsymbol{\lambda}$ is the solution of equation (6). The matrix $\partial \boldsymbol{A}/\partial \tilde{\varepsilon}_n$ is the analytical derivative of the system matrix with respect to the dielectric constant of the nth perturbed node. This matrix is different from its analogous matrix in equation (5) as it is not a full matrix. It is a sparse matrix with nonzero elements only at locations corresponding to the nth node links.

Figure 1*a* illustrates a possible TLM discretization of a lossy inhomogeneous structure. The structure is composed of three dielectric mediums. Consider first the case when x_k is $\tilde{\varepsilon}_2$, or ε_{r2}, or σ_2. Here, the exact sensitivity, equation (5), can be directly applied with the term $\partial \boldsymbol{A}/\partial x_k$ obtained analytically, i.e., using common rules of differentiation. The nonzero elements in $\partial \boldsymbol{A}/\partial x_k$ correspond to all the links of the related medium, i.e., medium 2, plus those at the boundary interfaces of medium 2 with medium 1 and medium 3 (see the arrowed links in Fig. 1b).

Now, consider another case when the thickness of the second medium W is the shape parameter of interest, i.e., we want to compute df/dW. Here, the exact sensitivity equation (5) can not be applied (refer to section III). Our semi-analytical equation (8), however, can be applied making use of the available $\partial \boldsymbol{A}/\partial \tilde{\varepsilon}$ values. In this case, a discrete perturbation of $\Delta W = \delta$ in W results in a derivative matrix $\partial \boldsymbol{A}/\partial \tilde{\varepsilon}_n$ with nonzero coefficients only at locations that correspond to the arrowed links shown in Fig. 1c. These are the links at the right-hand side of medium 2 affected by the discrete perturbation. The material change at these border cells is described mathematically in the sensitivity expression through the term $\left(\partial \boldsymbol{A}/\partial \tilde{\varepsilon}_n\right) \cdot \left(\partial \tilde{\varepsilon}_n/\partial x_k\right)$, see equation (8).

Note that equation (8) cannot be exact with structured grid solvers when x_k is related to a shape parameter such as W. This is because $\partial \tilde{\varepsilon}_n/\partial x_k$ is computed for each link of the nth perturbed node with $dx_k \approx \Delta x_k = \delta$ (stepwise change). Therefore, we approximate the analytical derivative $\partial \tilde{\varepsilon}_n/\partial x_k$ by its difference ratio $\Delta \tilde{\varepsilon}_n/\delta$, i.e., formula (8) becomes

$$\frac{df}{dx_k} = \frac{\partial f}{\partial x_k} + \boldsymbol{\lambda}^H \cdot \left[\frac{\partial \boldsymbol{V}_s}{\partial x_k} - \sum_{n=1}^{m} \left(\frac{\partial \boldsymbol{A}}{\partial \tilde{\varepsilon}_n} \frac{\Delta \tilde{\varepsilon}_n}{\delta} \right) \cdot \boldsymbol{v} \right], \tag{9}$$
$$k = 1, \ \ldots, \ K.$$

Despite this approximation, the semi-analytical sensitivity, equation (9) is still advantageous over equation (7) because the analytical derivative $\partial \boldsymbol{A}/\partial \tilde{\varepsilon}_n$ allows us to use the solution of the unperturbed problem (3). Thus, when the structure contains lossy dielectrics, we can use equation (9) and avoid the K approximations used in equation (7).

When x_k is related to a material parameter of the structure such as ε_r and σ, sensitivity equation (9) reduces to the exact equation (5).

Since all the elements of $\boldsymbol{A}$ are analytic functions of $\tilde{\varepsilon}$, see equations (1) to (2), the Cauchy-Riemann relations hold, allowing the use of $\tilde{\varepsilon}$, or ε_r, or σ in equation (9),

$$\frac{\partial \Re \boldsymbol{A}}{\partial \Re \tilde{\varepsilon}} = \frac{\partial \Im \boldsymbol{A}}{\partial \Im \tilde{\varepsilon}}, \quad \frac{\partial \Re \boldsymbol{A}}{\partial \Im \tilde{\varepsilon}} = -\frac{\partial \Im \boldsymbol{A}}{\partial \Re \tilde{\varepsilon}}. \tag{10}$$

Here, $\Re$ and $\Im$ denote the real and the imaginary part of the complex quantity, respectively. Consider, for example, the coefficients of the system matrix $\boldsymbol{A}$ that correspond to the links within the dielectric. According to the Cauchy-Riemann relations, equation (10), their analytical derivatives with respect to the constitutive parameters relate as

$$\begin{aligned} \frac{\partial \boldsymbol{A}}{\partial \varepsilon_r} &= \frac{\partial \Re \boldsymbol{A}}{\partial \varepsilon_r} + j\frac{\partial \Im \boldsymbol{A}}{\partial \varepsilon_r} \\ &= j\omega\varepsilon_0 \left[\frac{\partial \Im \boldsymbol{A}}{\partial \sigma} - j\frac{\partial \Re \boldsymbol{A}}{\partial \sigma} \right] \\ &= \omega\varepsilon_0 \left[\frac{\partial \Re \boldsymbol{A}}{\partial \sigma} + j\frac{\partial \Im \boldsymbol{A}}{\partial \sigma} \right] \\ &= \omega\varepsilon_0 \frac{\partial \boldsymbol{A}}{\partial \sigma}. \end{aligned} \tag{11}$$

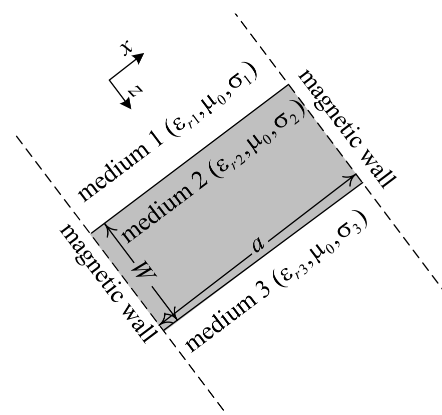

Fig. 2. 2D top view of the structure under study.

V. RESULTS AND DISCUSSION

The different AVM approaches discussed in sections III and IV are verified through the response sensitivity in a problem of a plane wave normally incident on a three-layer structure see Fig. 2. The structure is of width a = 30 mm and is discretized using a uniform 1 mm mesh cell, i.e., δ = 1 mm. The first layer of the structure, medium 1, has relative permittivity $\varepsilon_r = \varepsilon_{r1}$, permeability of free space μ_0, and conductivity $\sigma = \sigma_1$. The second layer, medium 2, is characterized by the constitutive parameters ε_{r2}, μ_0, σ_2, and is of thickness W. The third layer, medium 3, has the same constitutive parameters as medium 1, i.e., $\varepsilon_{r3} = \varepsilon_{r1}$, $\mu_3 = \mu_0$, and $\sigma_3 = \sigma_1$. The sensitivity of the response function f is measured with respect to variations in the vector of parameters $\boldsymbol{x} = [\varepsilon_{r2} \ \sigma_2 \ W]^T$ using applicable sensitivity expressions (5) to (9).

First, the response function f is defined as the complex electric field polarized in the y-direction

($f = E$) at the output port (medium 3). In this case, mediums 1 and 3 are set to be loss-free, i.e., $\sigma_1 = \sigma_2 = 0$, with $\varepsilon_{r1} = \varepsilon_{r3} = 1$. Medium 2 is lossy with nominal values $\varepsilon_{r2} = 5$ and $\sigma = 0.05$ S/m. The thickness is $W = 16$ mm. The sensitivities are computed at frequency 3.0 GHz.

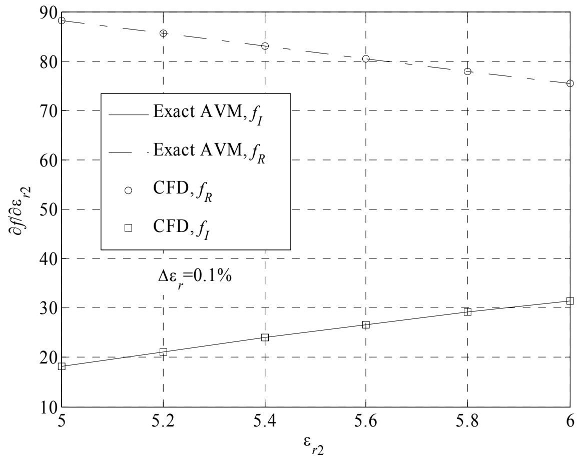

Fig. 3. Response sensitivity of $f = E$ with respect to ε_{r2} at the nominal value of $\tilde{\varepsilon}$, $f_R = \Re E$ and $f_I = \Im E$.

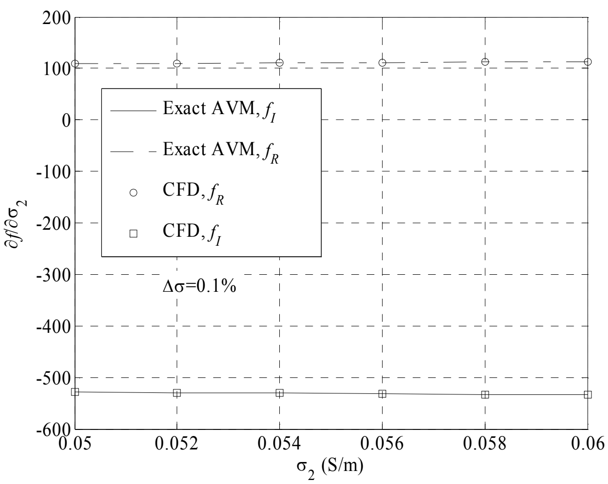

Fig. 4. Response sensitivity of $f = E$ with respect to σ_2 at the nominal value of $\tilde{\varepsilon}$, $f_R = \Re E$ and $f_I = \Im E$.

Figures 3 and 4 show the sensitivity of f with respect to the material parameters ε_{r2} and σ_2 computed using equation (5). These sensitivities are exact because the derivatives $\partial \boldsymbol{A}/\partial\varepsilon_{r2}$ and $\partial \boldsymbol{A}/\partial\sigma_2$ are analytic. Comparisons with response-level central finite difference (CFD) derivatives are also made using a finite perturbation of 0.1 percent. As expected, the agreement is excellent.

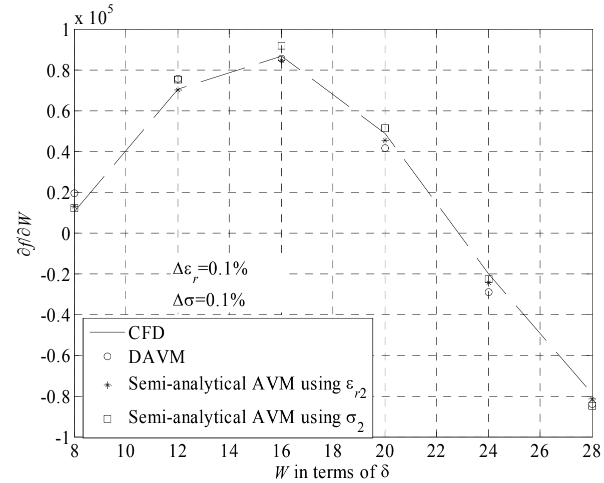

Fig. 5. Response sensitivity of $f_R = \Re E$ with respect to W at the nominal value of $\tilde{\varepsilon}$.

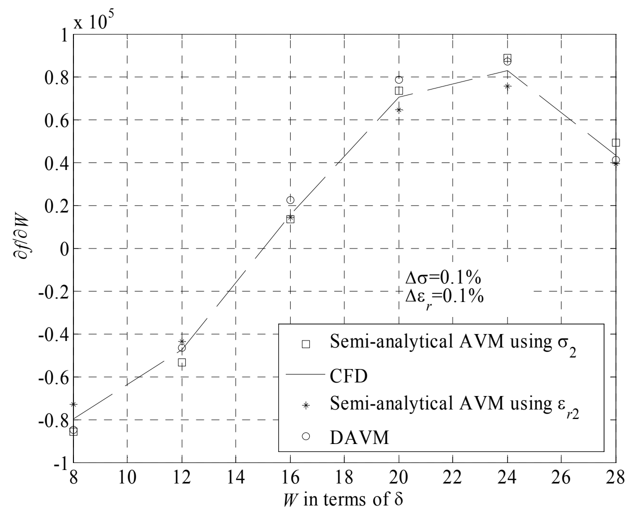

Fig. 6. Response sensitivity of $f_I = \Im E$ with respect to W at the nominal value of $\tilde{\varepsilon}$.

Figures 5 and 6 show the sensitivity of f with respect to the shape parameter W computed using: (i) response-level CFD; (ii) the DAVM sensitivity equation (7); (iii) the semi-analytical AVM equation (9) with $\partial \boldsymbol{A}/\partial\varepsilon_{r2}$ computed analytically and $d\varepsilon_{r2}/dW$ approximated as $\Delta\varepsilon_r/\delta$; and (iv) the semi-analytical AVM equation (9) with $\partial \boldsymbol{A}/\partial\sigma_2$ and $d\sigma_2/dW \approx \Delta\sigma_2/\delta$. The agreement is good.

Notice that for a complex objective function $f = f_R + jf_I$,[1] the derivative of the magnitude $|f|$ and phase ϕ can be easily extracted when $\partial f_R/\partial x_k$

[1] f_R and f_I are the real and imaginary parts of the complex function f, respectively.

and $\partial f_I / \partial x_k$ are known [12],

$$\frac{\partial |f|}{\partial x_k} = \frac{1}{|f|} \Re\left\{ f^* \cdot \frac{\partial f}{\partial x_k} \right\}, \quad (12)$$

and

$$\frac{\partial \phi}{\partial x_k} = \frac{1}{|f|^2} \Im\left\{ f^* \cdot \frac{\partial f}{\partial x_k} \right\}, \quad (13)$$

where

$$\frac{\partial f}{\partial x_k} = \frac{\partial f_R}{\partial x_k} + j \frac{\partial f_I}{\partial x_k}. \quad (14)$$

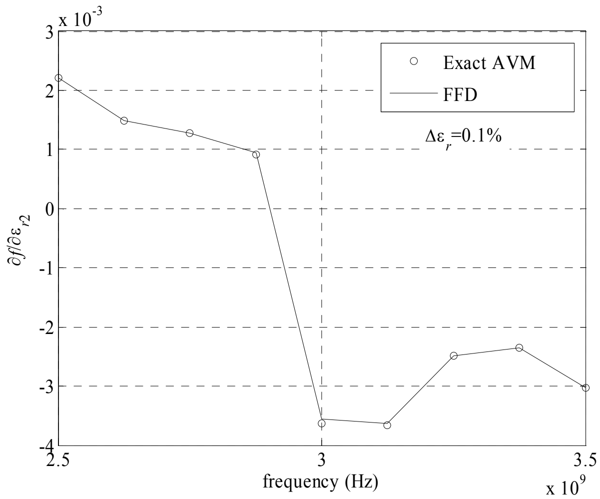

Fig. 7. Response sensitivity of $f = |\rho|^2$ with respect to ε_{r2} at nominal value of $\tilde{\varepsilon}$.

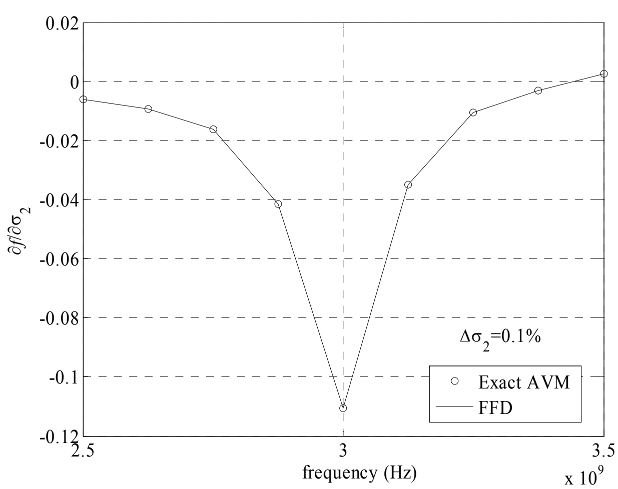

Fig. 8. Response sensitivity of $f = |\rho|^2$ with respect to σ_2 at nominal value of $\tilde{\varepsilon}$.

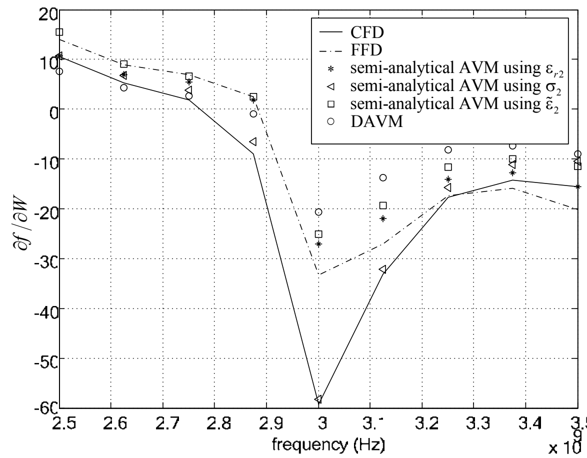

Fig. 9. Response sensitivity of $f = |\rho|^2$ with respect to W at the nominal value for $\tilde{\varepsilon}$.

For example, the sensitivity of $f = |\rho|^2$ where ρ is the reflection coefficient, is computed as $\partial |\rho|^2 / \partial x_k = 2\Re\{\rho^* \cdot \partial \rho / \partial x_k\}$.

We next verify the AVM approaches by computing the sensitivity of f when $f = |\rho|^2$. All three mediums in this case are lossy [see Fig. 2]. The first and third mediums have $\varepsilon_{r1} = \varepsilon_{r3}$ =15 and $\sigma_1 = \sigma_3$ = 0.15 S/m. The second medium has ε_{r2} = 80 and σ_2 = 4.0 S/m. The sensitivity is measured with respect to changes in the parameters of the second medium $\boldsymbol{x} = [\varepsilon_{r2} \;\; \sigma_2 \;\; W]^T$ for a frequency sweep from 2.5 GHz to 3.5 GHz.

Figures 7 and 8 show the sensitivity of f with respect to the constitutive parameters computed using the exact AVM equation (5). The results are also compared with response-level forward finite difference (FFD) estimates using a finite perturbation of 0.1 percent. Excellent agreement is obtained.

The sensitivity of f with respect to the shape parameter W is shown in Fig. 9. It is computed using: (i) response-level central and forward finite differences; (ii) the semi-analytical AVM equation (9) with $\partial \boldsymbol{A}/\partial \varepsilon_{r2}$ computed analytically and $\partial \varepsilon_{r2}/\partial W$ approximated as $\Delta\varepsilon_r/\delta$; (iii) the semi-analytical equation (9) with $\partial \boldsymbol{A}/\partial \sigma_2$ and $\partial \sigma_2/\partial W \approx \Delta\sigma_2/\delta$, (iv) the semi-analytical equation (9) with $\partial \boldsymbol{A}/\partial \tilde{\varepsilon}_2$ and $\partial \tilde{\varepsilon}_2/\partial W \approx \Delta\tilde{\varepsilon}/\delta$, and (vi) the DAVM equation (7).

A number of observations can be made with respect to Fig. 9. Firstly, we notice that the disparity between the forward and central response level sensitivities is noticeable. The difference comes as a result of the finite discretization step δ. A finer discretization may improve the accuracy, however, this is not necessarily so. This has always been considered the major drawback of response-level sensitivities.

Secondly, due to the semi-analytical nature of the newly proposed approach, the accuracy of the computed sensitivities can substantially improve compared to the preceding DAVM approach. This is provided that the right choice of the intermediate variable in the semi-analytical formula is made. For example, the sensitivities computed in case (ii) employed ε_{r2} as the intermediate variable, sensitivities computed in case (iii) employed σ_2 as the intermediate variable, and those computed in case (iv) employed $\tilde{\varepsilon}_2$ as the intermediate variable. In general, using $\tilde{\varepsilon}_2$ yields the most accurate results.

A final remark concerns a shape parameter, which relates to a region where $\omega\varepsilon_0\varepsilon_r \leq \sigma$, i.e., the shape parameter relates to a geometrical detail, which is a good conductor. The large values of σ cause the corresponding $\boldsymbol{A}$-matrix coefficients to almost vanish resulting in numerical errors. Hence, in this case, the DAVM technique is the only approach for sensitivity analysis with respect to such details. Obviously, this is also the case when the detail is a perfect conductor.

Thirdly, the disparity between the results computed using the semi-analytical approach with ε_{r2} and σ_2 as intermediate variables and those computed using $\tilde{\varepsilon}_2$ is also noticeable. This is a result of the nonlinearity of f, the discretization step size δ, and the fact that all three mediums are lossy with large contrast between their constitutive parameters. In general, when the adjacent mediums have very different constitutive parameters, it is recommended to use $\tilde{\varepsilon}$ in equation (9) and not ε_r or σ. That is because $\tilde{\varepsilon}$ takes into account the change in both the permittivity and the conductivity of the affected links at the same time.

There are cases where only ε_r can be used as the intermediate variable in the semi-analytical sensitivity expression. An example of such cases is when the adjacent mediums at a perturbed boundary have different permittivities but the same or no conductivity.

Alternatively, if the two adjacent mediums at a perturbed boundary have the same permittivity but different conductivities, only σ can be used as an intermediate variable.

Here, we make some general remarks with regard to the numerical sensitivity analysis with shape parameters. The accuracy of the estimated sensitivities (exact or discrete) with any numerical analysis technique can be only as good as that of the response function. The accuracy of the latter depends on the setup and the convergence of the numerical solution.

Also, the difference between the values of the constitutive parameters of the neighboring mediums is a factor affecting the accuracy of the computed sensitivities. This factor is interrelated with the discretization step size δ. In the case of the finite-difference estimation of the response sensitivity, these two factors influence the accuracy of the estimates through round-off errors. On one hand, smaller δ typically leads to improved accuracy of the numerical solution for the response calculation. On the other hand, a shape perturbation of one δ for adjacent mediums of very similar constitutive parameters may produce a response, which is practically identical to the response of the original unperturbed structure. In such a case, the finite-difference estimate is very unreliable. Possible remedy is a perturbation of several δ. There is, however, no generally valid prescription for a proper value of the perturbation with the finite-difference approach at the response level. This, together with the obvious numerical inefficiency of the finite-difference approach, is its main drawback.

For a given δ, the accuracy of our adjoint-based techniques is better or equivalent to that of the finite difference approaches because they are less prone to round-off errors. Consider, for example, the semi-analytical adjoint-based equation (9). The only difference term $\Delta\tilde{\varepsilon}$ is not affected by round-off errors because, in practice, when two materials are considered different, the difference in their constitutive parameters is far more significant than the machine error. For the same reason, with the DAVM, the different constitutive parameter values produce significantly different coefficients of the system matrix $\boldsymbol{A}$. Subsequently, the elements of the difference matrix $\Delta\boldsymbol{A}$ in equation (7) are never very small.

VI. CONCLUSIONS

For the first time, we present adjoint-based approaches for efficient sensitivity analysis of high-frequency structures developed for lossy inhomogeneous materials. The approaches are applicable in the case of material and shape parameters. The first approach is an approximate discrete adjoint-based approach adapted from our preceding technique. It was originally developed for lossless homogenous problems. The approach is investigated here for the first time with lossy inhomogeneous materials. It requires field approximations of the perturbed problems. We improve this approach by proposing a semi-analytical formula. The newly developed formula utilizes the analytical dependence of the system matrix elements on the constitutive parameters of the structure materials. This approach avoids the approximations needed in the preceding technique leading to simplification in the implementation and improvement in the accuracy. We show that the accuracy of the proposed semi-analytical approach is affected by the variation between the constitutive parameters of the neighboring mediums.

We also show that this effect can be reduced by using the complex permittivity as the intermediate variable in the semi-analytical expression.

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