MoM Solution to Scattering from Three-Dimensional Inhomogeneous Magnetic and Dielectric Bodies

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Abstract — This paper presents a method of moments solution to scattering problems that involve inhomogeneous magnetic and dielectric bodies of arbitrary shapes. The volume equivalence principle was used to switch from an original problem that deals with an inhomogeneous magnetic and dielectric scatterer to a problem in free space with equivalent sources. The problem is described through a mixed potential formulation. The method of moments technique is then applied to achieve a numerical solution to the original problem. The volume of the scatterer is meshed by tetrahedral cells and face-based functions are applied to expand unknown quantities. Special attention is paid to the curl operation on vector potentials and corresponding volume integrals. The proposed formulation has been evaluated through some examples.

I. INTRODUCTION

The three-dimensional approach in solving particular electromagnetic scattering problems using the method of moments has been applied for the first time in the early 1980's. In 1984 Schaubert et al. [1] used tetrahedral cells to calculate electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies. This was the first time that three-dimensional cells and the volume formulation have been used in the computation of scattering involving the method of moments. They have basically opened a door to a new area of inhomogeneous scatterers. In 1986 Schaubert and Meaney have improved the calculations [2], especially in the domain of computation time. Singular integrals resulting from integration in the vicinity of sources have been evaluated by isolating the singularity in an infinitesimally small sphere and then using the analytical approach described in [3]. Recent work by Kulkarni et al. published in 2004 [4] compared face-based expansion functions used in [1] to the edge-based solenoidal basis functions used in [5], [6], and [7].

An extensive literature in this area shows a continuous interest in the method of moments technique in solving numerous theoretical and practical electromagnetic problems related to electromagnetic scattering. Many electromagnetic scattering problems have been solved using this approach. Most of them, however, deal with the two-dimensional meshing and expansion functions. These problems usually involve homogeneous scatterers and the surface integral formulation [8]. Induced polarization currents, be them electric and/or magnetic, are located at the surface of the scatterer and can be represented through two-dimensional functions (pulse, rooftop, etc). There is not enough effort put in solving electromagnetic scattering from inhomogeneous scatterers. There are only a few papers that deal with inhomogeneous dielectrics. Usually they are related to some conventional geometries such as the sphere, the cube, and the cylinder. Most of them utilize symmetry of the shape in order to reach the final solution.

The main contribution of this paper is that it offers a generalized volume integral formulation for scatterers of arbitrary shape filled with an arbitrary magnetic and dielectric medium. The proposed formulation has been applied to numerous practical examples of scatterers illuminated by electromagnetic plane waves. Although this article presents data related to some symmetrical three dimensional scatterers, the approach is not limited to the shape of the scatterer in any sense. This is a significant generalization because previous work in this area dealt with scatterers of particular shapes. Furthermore, the developed solution does not put any limits on the geometrical assignment of material properties to the scatterer. It can be applied to multilayered scatterers, scatterers with materials assigned to different regions of the scatterer in a linear, exponential or any other fashion, etc.

II. FORMULATION

Assume that a source-free region containing an inhomogeneous magnetic and dielectric scatterer is illuminated by a plane electromagnetic wave. A magnetic and dielectric material is one that contains both magnetic and electric properties, i.e. both μ_r and ε_r of the material are not equal to 1. We can describe the electromagnetic behavior of the structure using Maxwell's equations

$$\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu_0\mu_r(\mathbf{r})\,\mathbf{H}(\mathbf{r})\,, \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = j\omega\varepsilon_0\varepsilon_r(\mathbf{r})\,\mathbf{E}(\mathbf{r})\,, \tag{2}$$

$$\nabla \cdot [\varepsilon_0\varepsilon_r(\mathbf{r})\,\mathbf{E}(\mathbf{r})] = 0\,, \tag{3}$$

$$\nabla \cdot [\mu_0\mu_r(\mathbf{r})\,\mathbf{H}(\mathbf{r})] = 0 \tag{4}$$

where $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ are the complex valued phasors of the electric and magnetic fields respectively. We replace

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the inhomogeneous scatterer by equivalent volume polarization currents $\mathbf{J}(\mathbf{r})$ and $\mathbf{M}(\mathbf{r})$ and charges $\rho_e(\mathbf{r})$ and $\rho_m(\mathbf{r})$ given by

$$\mathbf{J}(\mathbf{r}) = j\omega\varepsilon_0[\varepsilon_r(\mathbf{r})-1]\mathbf{E}(\mathbf{r}), \quad (5)$$

$$\mathbf{M}(\mathbf{r}) = j\omega\mu_0[\mu_r(\mathbf{r})-1]\mathbf{H}(\mathbf{r}), \quad (6)$$

$$\rho_e(\mathbf{r}) = \varepsilon_0\varepsilon_r(\mathbf{r})\,\mathbf{E}(\mathbf{r})\cdot\nabla\left(\frac{1}{\varepsilon_r(\mathbf{r})}\right), \quad (7)$$

$$\rho_m(\mathbf{r}) = \mu_0\mu_r(\mathbf{r})\,\mathbf{H}(\mathbf{r})\cdot\nabla\left(\frac{1}{\mu_r(\mathbf{r})}\right) \quad (8)$$

and switch from the initial problem that involves inhomogeneity to a simpler problem that involves equivalent sources in free space. In (7) and (8), $\rho_e(\mathbf{r})$ and $\rho_m(\mathbf{r})$ were obtained by using the equations of continuity

$$\rho_e(\mathbf{r}) = -\frac{1}{j\omega}\nabla\cdot\mathbf{J}(\mathbf{r}) \quad (9)$$

$$\rho_m(\mathbf{r}) = -\frac{1}{j\omega}\nabla\cdot\mathbf{M}(\mathbf{r}). \quad (10)$$

The scattered field ($\mathbf{E}^s$, $\mathbf{H}^s$) can be obtained through the mixed potential formulation, which is

$$\mathbf{E}^s(\mathbf{r}) = -j\omega\mathbf{A}(\mathbf{r}) - \nabla V(\mathbf{r}) - \nabla\times\frac{\mathbf{F}(\mathbf{r})}{\varepsilon_0}, \quad (11)$$

$$\mathbf{H}^s(\mathbf{r}) = -j\omega\mathbf{F}(\mathbf{r}) - \nabla U(\mathbf{r}) + \nabla\times\frac{\mathbf{A}(\mathbf{r})}{\mu_0} \quad (12)$$

where $\mathbf{A}(\mathbf{r})$ and $\mathbf{F}(\mathbf{r})$ are the vector potential functions and $U(\mathbf{r})$ and $V(\mathbf{r})$ are the scalar potential functions:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_T \mathbf{J}(\mathbf{r}')\frac{\exp(-jk|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}d\tau', \quad (13)$$

$$\mathbf{F}(\mathbf{r}) = \frac{\varepsilon_0}{4\pi}\int_T \mathbf{M}(\mathbf{r}')\frac{\exp(-jk|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}d\tau', \quad (14)$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_T \rho_e(\mathbf{r}')\frac{\exp(-jk|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}d\tau', \quad (15)$$

$$U(\mathbf{r}) = \frac{1}{4\pi\mu_0}\int_T \rho_m(\mathbf{r}')\frac{\exp(-jk|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}d\tau' \quad (16)$$

where T is the volume of the scatterer from the initial problem, $d\tau'$ is the differential element of volume at $\mathbf{r}'$, $\mathbf{r}'$ is the position vector of the source point, and $\mathbf{r}$ is the position vector of the observation point.

Replacing $\mathbf{E}$ in (5) by $\mathbf{E}^s+\mathbf{E}^{inc}$, replacing $\mathbf{H}$ in (6) by $\mathbf{H}^s+\mathbf{H}^{inc}$ where ($\mathbf{E}^{inc}$,$\mathbf{H}^{inc}$) is the incident field, substituting (11) and (12) into the resulting equations, and using

$$\mathbf{J}(\mathbf{r}) = j\omega\frac{\varepsilon_r(\mathbf{r})-1}{\varepsilon_r(\mathbf{r})}\mathbf{D}'(\mathbf{r}) \quad (17)$$

$$\mathbf{M}(\mathbf{r}) = j\omega\frac{\mu_r(\mathbf{r})-1}{\mu_r(\mathbf{r})}\mathbf{B}'(\mathbf{r}) \quad (18)$$

to eliminate $\mathbf{J}(\mathbf{r})$ and $\mathbf{M}(\mathbf{r})$ in favor of new unknowns $\mathbf{D}'$ and $\mathbf{B}'$, we obtain

$$j\omega\mathbf{A} + \nabla V + \nabla\times\frac{\mathbf{F}}{\varepsilon_0} + \frac{1}{\varepsilon_0\varepsilon_r}\mathbf{D}' = \mathbf{E}^{inc} \quad (19)$$

$$j\omega\mathbf{F} + \nabla U - \nabla\times\frac{\mathbf{A}}{\mu_0} + \frac{1}{\mu_0\mu_r}\mathbf{B}' = \mathbf{H}^{inc}. \quad (20)$$

As it is related to $\mathbf{J}$ in (17), $\mathbf{D}'$ reduces to the displacement vector $\mathbf{D}=\varepsilon_0\varepsilon_r\mathbf{E}$ when $\mathbf{J}$ satisfies (19). Similarly, $\mathbf{B}'$ reduces to the magnetic induction $\mathbf{B}=\mu_0\mu_r\mathbf{H}$ when $\mathbf{H}$ satisfies (20).

III. APPLYING THE METHOD OF MOMENTS TECHNIQUE

Equations (19) and (20) are two equations that cannot be solved analytically. Expanding the unknown quantities $\mathbf{D}'(\mathbf{r})$ and $\mathbf{B}'(\mathbf{r})$ in terms of a set of face-based functions $\{\mathbf{f}_n(\mathbf{r})\}$ on a tetrahedral mesh as described in [1] and testing with $\{\mathbf{f}_m(\mathbf{r})\}$ as in Galerkin's method, (19) and (20) are transformed into the following set of equations:

$$j\omega\langle\mathbf{f}_m,\mathbf{A}\rangle + \langle\mathbf{f}_m,\nabla V\rangle + \left\langle\mathbf{f}_m,\nabla\times\frac{\mathbf{F}}{\varepsilon_0}\right\rangle + \left\langle\mathbf{f}_m,\frac{1}{\varepsilon_0\varepsilon_r}\mathbf{D}'\right\rangle = \langle\mathbf{f}_m,\mathbf{E}^{inc}\rangle \quad (21)$$

$$j\omega\langle\mathbf{f}_m,\mathbf{F}\rangle + \langle\mathbf{f}_m,\nabla U\rangle - \left\langle\mathbf{f}_m,\nabla\times\frac{\mathbf{A}}{\mu_0}\right\rangle + \left\langle\mathbf{f}_m,\frac{1}{\mu_0\mu_r}\mathbf{B}'\right\rangle = \langle\mathbf{f}_m,\mathbf{H}^{inc}\rangle \quad (22)$$

where $\mathbf{D}'$ and $\mathbf{B}'$ are linear combinations of the $\mathbf{f}_n$'s containing unknown coefficients D_n and B_n and, for arbitrary $\mathbf{A}$ and $\mathbf{B}$, $\langle\mathbf{A},\mathbf{B}\rangle$ is the symmetric product of $\mathbf{A}$ and $\mathbf{B}$ defined to be the integral of their dot product over the volume of the scatterer. Equations (21) and (22) can be solved numerically for $\{D_n\}$ and $\{B_n\}$. All the kinds of terms in (21) and (22) except those involving the curls of vector potentials are treated in [1], [3], and [9].

Consider the term involving the curl of the magnetic vector potential in (22). The vector potential $\mathbf{A}$ is given by (13) where $\mathbf{J}$ is given by (17) with

$$\mathbf{D}'(\mathbf{r}) = \sum_{n=1}^{N} D_n\mathbf{f}_n(\mathbf{r}) \quad (23)$$

where

$$\mathbf{f}_n(\mathbf{r}) = \begin{cases} \dfrac{s_n}{3W_n^+}\boldsymbol{\rho}_n^+, & \mathbf{r}\in T_n^+ \\ \dfrac{s_n}{3W_n^-}\boldsymbol{\rho}_n^-, & \mathbf{r}\in T_n^- \end{cases}. \quad (24)$$

In (24), T_n^+ and T_n^- are the two tetrahedrons that have the same face s_n, $W_n^{\pm}$ is the volume of $T_n^{\pm}$, $\boldsymbol{\rho}_n^+$ is the vector from the vertex of T_n^+ opposite s_n to $\mathbf{r}$, and $\boldsymbol{\rho}_n^-$ is the vector from $\mathbf{r}$ to the vertex of T_n^- opposite s_n (Figure 1).

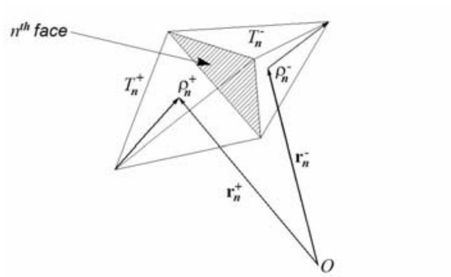

Fig. 1. Tetrahedral elements T_n^+ and T_n^- and notation.

Using (13), (17), (23), and (24), and assuming that ε_r is constant in each tetrahedron, we obtain

$$\frac{1}{\mu_0}\nabla\times\mathbf{A}(\mathbf{r}) = j\omega\sum_{n=1}^{N} D_n\left[\frac{\beta_{1n}^{+} s_n}{3}\nabla\times\mathbf{I}_{1n}^{+}(\mathbf{r}) + \frac{\beta_{1n}^{-} s_n}{3}\nabla\times\mathbf{I}_{1n}^{-}(\mathbf{r})\right] \quad (25)$$

where

$$\mathbf{I}_{1n}^{\pm}(\mathbf{r}) = \frac{1}{W_n^{\pm}}\int_{T_n^{\pm}}\boldsymbol{\rho}_n^{\pm}\frac{\exp(-jkR)}{4\pi R}d\tau' \quad (26)$$

and

$$\beta_{1n}^{\pm} = \frac{\varepsilon_{rn}^{\pm}-1}{\varepsilon_{rn}^{\pm}} \quad (27)$$

where $\varepsilon_{rn}^{\pm}$ is ε_r in $T_n^{\pm}$. Testing (25) with the function $\mathbf{f}_m(\mathbf{r})$ yields

$$\begin{aligned}\left\langle \mathbf{f}_m(\mathbf{r}),\nabla\times\frac{\mathbf{A}(\mathbf{r})}{\mu_0}\right\rangle &= \int_{T_m^+}\mathbf{f}_m(\mathbf{r})\cdot\left[\nabla\times\frac{\mathbf{A}(\mathbf{r})}{\mu_0}\right]d\tau \\ &+ \int_{T_m^-}\mathbf{f}_m(\mathbf{r})\cdot\left[\nabla\times\frac{\mathbf{A}(\mathbf{r})}{\mu_0}\right]d\tau \\ &= \frac{j\omega}{3}\sum_{n=1}^{N} s_n D_n\left[\beta_{1n}^{+} I_{3n}\left(T_{m,n}^{+,+}\right) + \beta_{1n}^{-} I_{3n}\left(T_{m,n}^{+,-}\right)\right] \\ &+ \frac{j\omega}{3}\sum_{n=1}^{N} s_n D_n\left[\beta_{1n}^{+} I_{3n}\left(T_{m,n}^{-,+}\right) + \beta_{1n}^{-} I_{3n}\left(T_{m,n}^{-,-}\right)\right]\end{aligned} \quad (28)$$

where

$$I_{3n}\left(T_{m,n}^{+,\pm}\right) = \int_{T_m^+}\mathbf{f}_m(\mathbf{r})\cdot\nabla\times\mathbf{I}_{1n}^{\pm}(\mathbf{r})\,d\tau \quad (29)$$

$$I_{3n}\left(T_{m,n}^{-,\pm}\right) = \int_{T_m^-}\mathbf{f}_m(\mathbf{r})\cdot\nabla\times\mathbf{I}_{1n}^{\pm}(\mathbf{r})\,d\tau\,. \quad (30)$$

A similar derivation can be obtained for the term involving the curl of electric vector potential $\mathbf{F}$.

Let us now consider one of the integrals in (29):

$$\begin{aligned}I_{3n}\left(T_{m,n}^{+,+}\right) &= -\int_{T_m^+}\nabla\cdot\left(\mathbf{f}_m\times\mathbf{I}_{1n}^{+}\right)d\tau + \int_{T_m^+}\mathbf{I}_{1n}^{+}\cdot\left(\nabla\times\mathbf{f}_m\right)d\tau \\ &= -\sum_{i=1}^{4}\int_{s_{m,i}^+}\mathbf{n}_i^{+}\cdot\left(\mathbf{f}_m\times\mathbf{I}_{1n}^{+}\right)ds \\ &= -\frac{s_m}{3W_m^{+}}\sum_{i=1}^{4}\int_{s_{m,i}^+}\mathbf{I}_{1n}^{+}\cdot\left(\mathbf{n}_i^{+}\times\boldsymbol{\rho}_m^{+}\right)ds\end{aligned} \quad (31)$$

where $s_{m,i}^+$, i=1,…,4 are the four faces of the tetrahedron T_m^+ and $\mathbf{n}_i^+$ is the outward pointing unit normal vector to the face $s_{m,i}^+$.

If we now assume a fine mesh, then each of the four faces $s_{m,i}^+$, i=1,…,4 is so small that the integral $\mathbf{I}_{1n}^+(\mathbf{r})$ for $\mathbf{r}\in s_{m,i}^+$ can be approximated by its value at the centroid of $s_{m,i}^+$ at $\mathbf{r}=\mathbf{r}_{m,i}^{c+}$. Hence, we can write

$$\begin{aligned}I_{3n}\left(T_{m,n}^{+,+}\right) &= -\frac{s_m}{3W_m^{+}}\sum_{i=1}^{4}\int_{s_{m,i}^+}\mathbf{I}_{1n}^{+}\cdot\left(\mathbf{n}_i^{+}\times\boldsymbol{\rho}_m^{+}\right)ds \\ &= -\frac{s_m}{3W_m^{+}}\sum_{i=1}^{4}\mathbf{I}_{1n}^{+}\left(\mathbf{r}_{m,i}^{c+}\right)\cdot\int_{s_{m,i}^+}\left(\mathbf{n}_i^{+}\times\boldsymbol{\rho}_m^{+}\right)ds\,.\end{aligned} \quad (32)$$

Similar derivations can be performed for the other three integrals in (29) and (30).

Please note that the singularity resulting from $\mathbf{r}'=\mathbf{r}$ encountered in the evaluation of I_{3n} appears in expression (26) for $\mathbf{I}_{1n}^{\pm}$. So treating the singularity in expression (26) for $\mathbf{I}_{1n}^{\pm}$ automatically takes care of it in the evaluation of I_{3n}.

IV. NUMERICAL RESULTS

Numerical data obtained through the MATLAB implementation of the proposed formulation is given here. We considered two shapes of scatterers - a sphere and a cube. These bodies are illuminated by a θ-polarized plane electromagnetic wave incident from the direction where θ=180° and ϕ=0° ($\mathbf{E}^{inc}=-\hat{\mathbf{a}}_x E^{inc}e^{-jkz}$). Results presented here are co-polarized and cross-polarized bistatic radar cross sections.

We first investigated an inhomogeneous two-layer magnetic and dielectric sphere of radius R with $k_0R= k_0r_2$ =0.408 where k_0 is the free-space wave number. The radius of the core is half the radius of the whole sphere. The two layers of the sphere are assigned different material properties. As a first step in developing a mesh the entire outer surface of the sphere had been approximated by a grid of 72 triangles that served as a starting point for a tetrahedral mesh. Then a tetrahedral mesh has been grown from the outer triangular surface into the sphere producing a total of 256 tetrahedra and 548 faces. In order to achieve a better accuracy of the numerical results an additional refinement of the mesh in the close proximity to the outer surface has been undertaken, thus increasing the total number of tetrahedra to 520 and faces to 1184. Also, an additional refinement on the surface between the two layers with two different material properties has been performed. This surface contains surface charge and plays an extremely important role in the way how this electromagnetic environment behaves. Refining the mesh in the close proximity to this surface resulted in a total of 928 tetrahedra and 2000 faces. Finally, the radius of the sphere has been so adjusted that the total volume of the tetrahedral approximation of the sphere is equal to the actual volume of the initial sphere. All computations have been performed on a regular PC machine with a 64bit CPU and 1024MB of RAM. It took about 2 hours to perform all necessary computations and about 80MB to store all data of interest.

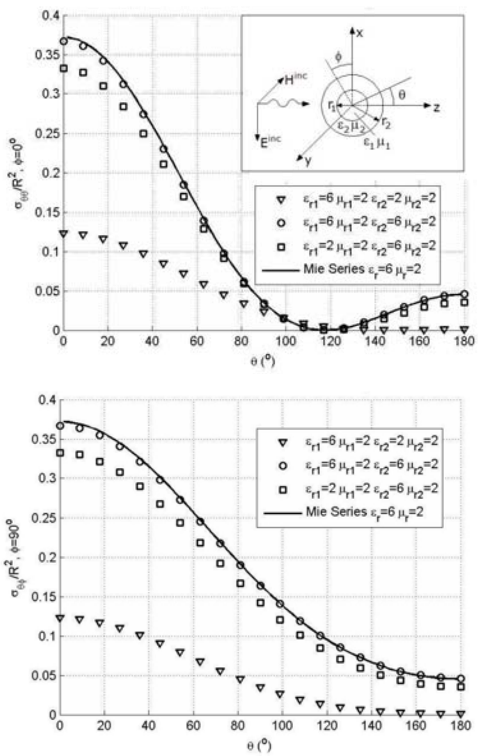

Fig. 2. Bistatic radar cross sections of an inhomogeneous two-layered dielectric and magnetic sphere for $k_0R=0.408$, various values of ε_r and μ_r, and number of tetrahedra=928.

Numerical results obtained for this scattering model are compared to results derived from the Mie series expansion and given in [10]. As observed in Figure 2, there is a good agreement between the numerical data and the Mie series solution. We have also performed a convergence test where we increased the number of tetrahedral mesh cells. It has been observed that an increase in the total number of meshing cells and decrease in the cell size increases the accuracy of the numerical solution and that MoM computation results converge to the exact results. We have varied relative permeabilities and permittivities of the two layers and investigated the scattering from this body. Please observe that a change in the relative permeability parameter generally affects the radar cross section of the magnetic and dielectric scatterer. The inner spherical layer plays a less important role in this effect, as it is partially shielded by the outer spherical shell and is electrically smaller than the outer layer [11].

We also investigated an inhomogeneous magnetic and dielectric cube assigned different electric and magnetic properties in a chess-like pattern (black cubical cells are

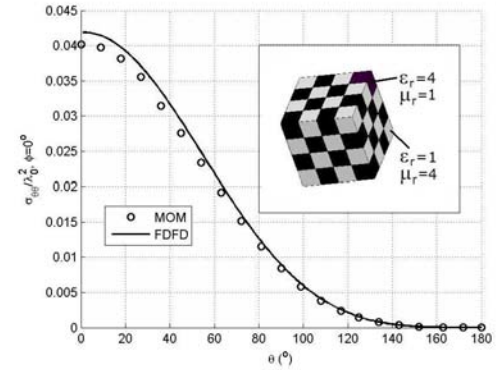

Fig. 3. Bistatic radar cross section $\sigma_{\theta\theta}$ of an inhomogeneous chess-like dielectric and magnetic cube (black inclusions $\varepsilon_r=4$ and $\mu_r=1$ and white inclusions $\varepsilon_r=1$ and $\mu_r=4$) for $d=0.2\lambda_0$ and number of tetrahedra=768.

filled by a dielectric with relative permeability $\varepsilon_r=4$ and white cubical cells are filled by a magnetic with relative permittivity $\mu_r=4$). The length of a side of the cube is $d=0.2\lambda_0$ where $\lambda_0=2\pi k_0$. The center of the cube coincides with the origin of the coordinate system. Numerical results for this scattering model are given in Figure 3 and compared to the results obtained through the implementation of the FDFD formulation presented in [12]. As we can see there was a good agreement between our solution based on the MoM approach and results based on FDFD approach.

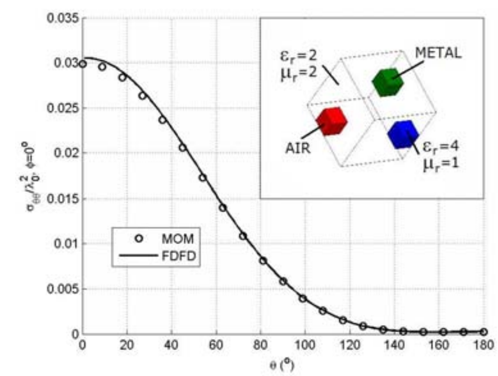

Fig. 4. Bistatic radar cross section $\sigma_{\theta\theta}$ of an inhomogeneous dielectric and magnetic cube with several inclusions (dielectric, air, metal) for $d=0.2\lambda_0$ and number of tetrahedra=768.

The proposed MoM formulation is then applied to calculate co-polarized and cross-polarized bistatic radar cross-sections of a magnetic and dielectric cube with several inclusions. Figure 4 presents numerical results obtained for the case of an inhomogeneous magnetic and dielectric cube ($\varepsilon_r=2$, $\mu_r=2$) with the length of a side $d=0.2\lambda_0$ filled with several inclusions (metal, for $-0.05\lambda_0 < x < 0$, $0 < y < 0.05\lambda_0$, $0.05\lambda_0 < z < 0.1\lambda_0$; air, for $-0.05\lambda_0 < x < 0$, $-0.1 < y < -0.05\lambda_0$, $-0.1\lambda_0 < z < -0.05\lambda_0$; dielectric $\varepsilon_r=4$, for $0.05\lambda_0 < x < 0.1\lambda_0$, $0.05\lambda_0 < y < 0.1\lambda_0$, $-0.05\lambda_0 < z < 0$). Results for cross-polarized bistatic radar cross section of this scatterer have also been compared to results obtained through the implementation of [12] and a good agreement was observed.

V. CONCLUSION

This paper presents a numerical solution based on the method of moments for electromagnetic scattering from arbitrarily shaped three-dimensional inhomogeneous magnetic and dielectric scatterers. Cases that we studied here are a dielectric and magnetic sphere and a dielectric and magnetic cube. As observed, the radar cross sections change when the permittivity or permeability of the scatterer change.

As noted we have used the volume equivalence principle. There is also the surface equivalence theorem. Consequently, surface integral equations may be derived from the surface equivalence principle and volume integral equations may be derived from the volume equivalence principle. The volume equivalence principle and corresponding volume integral equations that we used in our work have a number of advantages including the applicability to inhomogeneous scatterers and a better accuracy at resonances (compared to the surface approach). The volume equivalence principle and volume integral equations are therefore mostly used in problems involving penetrable inhomogeneous scatterers. Inhomogeneity as an essential property of a scatterer can not be entirely described by sources on its surface. That is why we need to put equivalent induced current and charge sources inside the scatterer and not only on its surface. This way sources become volumetric in their nature. As it can be observed we move from an original problem involving inhomogeneity to a problem involving induced sources in free space. The latter problem is much easier to be solved. Therefore, as a conclusion, the advantage of using the volume approach and volume integral equations over the surface approach and surface integral equations is their more general property and ability to deal with scatterers that are not homogeneous.

The proposed solution is applicable to any shape of scatterer and to any kind of spatial dependence of material properties. However, it may suffer from a rapid growth of computational complexity in the case of electrically large objects with increased mesh resolution. This problem may be avoided by reducing the total number of unknowns through a choice of different basis functions. Edge-based expansion functions, often referred to as three-dimensional solenoidal expansion functions are first proposed by [5] as a solution to this problem. They have a better convergence rate and higher numerical stability according to [4]. Another way to improve the efficiency of the developed algorithm is to use some acceleration techniques such as those based on the fast Fourier transform [13]. Additional research with the objective of increasing the efficiency of the computer program through a reduction of memory and time requirements is definitely worthwhile.

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