Robust Adaptive Beamforming Using Least Mean Mixed Norm Algorithm

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Abstract – This paper proposes an accurate and rapidly-convergent algorithm for enhanced adaptive beamforming based on the combination of the least mean mixed norm (LMMN) algorithm with initialization using sample matrix inversion (SMI). The algorithm uses a mixing parameter δ which controls the proportions of the error norms and offers an extra degree of freedom within the adaptation. Monte Carlo simulations show that the misadjustment curve has a minimum at $\delta = 0.40$ which means that the proposed algorithm has an optimum steady-state performance at this mixing parameter value. The convergence of the algorithm is further improved by employing SMI to initialize the weights vector in the LMMN update equation. This makes the proposed SMI-initialized LMMN algorithm have a better steady state performance when compared to the least mean squares (LMS) algorithm and better stability properties when compared to the least mean fourth (LMF) algorithm. Simulation results obtained show that the developed SMI-initialized LMMN algorithm outperforms other algorithms in terms of computational efficiency, numerical accuracy, and cosnvergence rate.

Keywords: Smart antennas, adaptive beamforming, and least mean squares.

I. INTRODUCTION

Smart antennas have emerged as one of the leading innovations for achieving highly efficient networks that maximize capacity and improve quality and coverage. Smart antennas provide greater capacity and performance benefits than standard antennas because they can be used to customize and fine-tune antenna coverage patterns to the changing traffic or radio frequency (RF) conditions in a wireless network [1]. Figure 1 shows a smart antenna system which consists of a uniform linear array (ULA) for which the current amplitudes are adjusted by a set of complex weights using an adaptive beamforming algorithm. The adaptive beamforming algorithm optimizes the array output beampattern such that maximum radiated power is produced in the directions of desired mobile users and deep nulls are generated in the directions of undesired signals representing co-channel interference from mobile users in adjacent cells. Prior to adaptive beamforming, the directions of users and interferers must be obtained using a direction-of-arrival (DOA) estimation algorithm [2].

Recent research efforts into smart antennas have varied over a wide range of methods and applications including array pattern synthesis based on null steering and multi-user beamforming using a phase control method [3], circular and hexagonal array geometries for smart antenna systems [4], adaptive and a switched beam smart antenna systems for wireless communications [5], tapered beamforming method for uniform circular arrays [6], beam steering with null and excitation constraints for linear antenna arrays [7], displaced sensor array for improved signal detection [8], and finally robust adaptive beamforming algorithms [9, 10]. The emphasis of this paper is on the development of enhanced adaptive beamforming algorithms for robust interference suppression.

Adaptive beamforming is achieved using adaptive antenna array for which the weights of the array element currents are adjusted in order to filter out the interfering signals from undesired sources, while enhancing the signal of interest from the desired source. Adaptive beamforming algorithms are typically characterized in terms of their convergence properties and computational complexity. One practical adaptive algorithm is the Least Mean Squares (LMS) which is simple to implement. It does not require measurements of the pertinent correlation functions, nor does it require matrix inversion. However, the LMS algorithm converges slowly when compared with other complicated algorithms such as the Recursive Least Square (RLS) [11]. On the other hand, Sample Matrix Inversion (SMI) algorithm has a fast convergence behavior However, because its speedy convergence is achieved through the use of matrix inversion, the SMI algorithm is computationally intensive. Moreover, the SMI algorithm has a block adaptive approach for which it is required that the signal environment does not undergo significant change during the course of block acquisition.

Various adaptive MMSE receivers are based on the standard quadratic cost function. So far, the LMS algorithm has proved popular for many applications because of its simplicity and ease of implementation [2, 12] . However, many alternatives can also be defined to improve the adaptation performance in specific statistical environments including the Least Mean Mixed Norm (LMMN) algorithm [13, 14]. This algorithm has been used to

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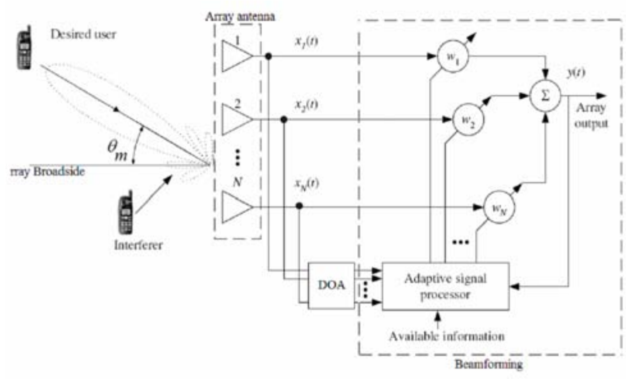

Fig. 1. A functional block diagram of a smart antenna system.

update the tap coefficients of the feedforward and feedback filters for the adaptation of the non-linear receiver, coupled with a second order phase tracking subsystem, for asynchronous DS-CDMA communication system impaired by double-spread multipath channel and Gaussian mixture impulsive noise [15]. The purpose of this paper is to develop an enhanced adaptive beamforming algorithm based on LMMN but with SMI initialization to ensure faster convergence. It is shown that judicious choice of the LMMN algorithm mixing parameter provides an algorithm with intermediate performance between the two special cases of least mean squares (LMS) and least mean fourth (LMF) algorithms. It is shown that the developed LMMN algorithm along with SMI initialization provides better steady state performance than the LMS algorithm and better stability properties than the LMF algorithm.

The rest of the paper is organized as follows: Section II describes the signal model for an adaptive beamformer based on the ULA configuration. Section III presents the theory of adaptive beamforming using the LMS algorithm, the LMF algorithm, and the proposed SMI-initialized LMMN algorithm. Simulation results are presented in Section IV showing that the proposed SMI-initialized algorithm outperforms the other algorithms. Finally, conclusions are given in Section V.

II. SIGNAL MODEL

The standard array geometry that has been used for smart antenna systems is the uniform linear array (ULA) depicted in Fig. 2. A uniform linear array consists of N elements that are spaced apart by half wavelength ($d = \lambda/2$). The inter-element spacing in a ULA is chosen to be $\lambda/2$ in order to reduce mutual coupling effects which deteriorate the performance of the DOA estimation algorithm as demonstrated in [16-21]. If the inter-element spacing ischosen to be smaller than $\lambda/2$, mutual coupling effects then cannot be ignored and the DOA estimation algorithm fails to produce the desired peaks in the angular spectrum. On the other hand, increasing the inter-element spacing beyond $\lambda/2$ results in spatial aliasing which takes the form of unwanted or misplaced peaks in the angular spectrum. It is therefore concluded that $d = \lambda/2$ represents the optimum value for the inter-element spacing in a ULA.

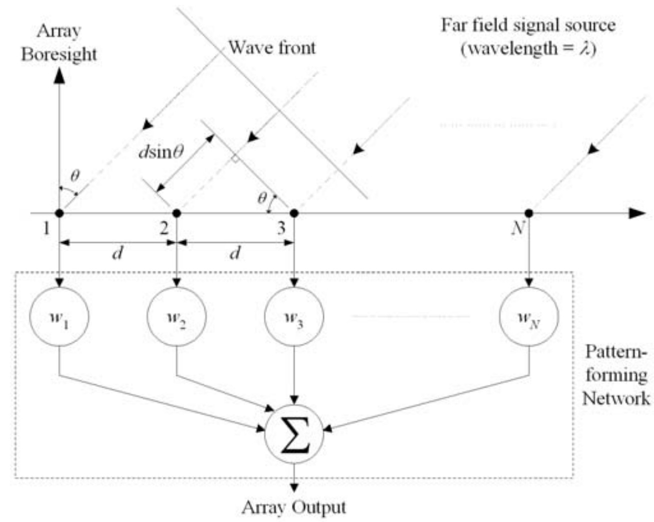

Fig. 2. Gemetry of a Uniform Linear Array (ULA) of N sensors that are equally spaced apart a distance $d = \lambda/2$..

The main advantage of using a ULA is that it has the simplest geometry, an excellent directivity, and produces the narrowest main-lobe in a given direction in comparison to other array geometries. The ULA consists of N linear equispaced omnidirectional sensors with inter-element spacing $d = \lambda/2$ and is positioned along the x axis with an azimuth angle θ_m measured with respect to the z axis. It is assumed that the ULA receives M narrowband source signals $s_m(t)$ from incidence directions $\theta_1, \theta_2, ...\theta_M$, as shown in Fig. 2. The array also receives I narrowband source signals $s_i(t)$ from undesired (or interference) users arriving at directions $\theta_1, \theta_2, ...\theta_I$. At a particular instant of time $t = 1, 2, ..., K$,where K is the total number of snapshots taken, the desired users signal vector $\mathbf{x}_S(t)$ can be defined as,

$$\mathbf{x}_S(t) = \sum_{m=1}^{M} \mathbf{a}(\theta_m) s_m(t) \tag{1}$$

where $\mathbf{a}(\theta_m)$ is the $N \times 1$ array steering vector which represents the array response at direction θ_m and is given by,

$$\mathbf{a}(\theta_m) = [\exp[j(n-1)\psi_m]^T; \qquad 1 \le n \le N \tag{2}$$

where$[(.)]^T$ is the transposition operator, and ψ_m represents the electrical phase shift from element to element along the array defined by $\psi_m = 2\pi(d/\lambda)\sin\theta_m$ where d is the inter-element spacing and λ is the wavelength of the received signal. The desired users signal vector $\mathbf{x}_S(t)$ of equation (1) can be written as,

$$\mathbf{x}_S(t) = \mathbf{A}_S \mathbf{s}(t) \tag{3}$$

where $\mathbf{A}_S$ is the $N \times M$ matrix of the desired users signal direction vectors and is given by,

$$\mathbf{A}_S = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ..., \mathbf{a}(\theta_M)] \tag{4}$$

and $\mathbf{s}(t)$ is the $M \times 1$ desired users source waveform vector defined as,

$$\mathbf{s}(t) = \begin{bmatrix} s_1(t) & s_2(t) & .. & .. & s_M(t) \end{bmatrix}^T. \tag{5}$$

We also define the undesired (or interference) users signal vector $\mathbf{x}_I(t)$ as,

$$\mathbf{x}_I(t) = \mathbf{A}_I \mathbf{i}(t) \tag{6}$$

where $\mathbf{A}_I$ is the $N \times I$ matrix of the undesired users signal direction vectors and is given by,

$$\mathbf{A}_I = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ..., \mathbf{a}(\theta_I)] \tag{7}$$

and $\mathbf{i}(t)$ is the $I \times 1$ undesired (or interference) users source waveform vector defined as,

$$\mathbf{i}(t) = \begin{bmatrix} i_1(t) & i_2(t) & .. & .. & i_I(t) \end{bmatrix}^T. \tag{8}$$

The overall received signal vector $\mathbf{x}(t)$ is given by the superposition of the desired users signal vector $\mathbf{x}_S(t)$, undesired (or interference) users signal vector $\mathbf{x}_I(t)$, and an $N \times 1$ vector $\mathbf{n}(t)$ which represents white Gaussian sensor noise. Hence, $\mathbf{x}(t)$ can be written as,

$$\mathbf{x}(t) = \mathbf{x}_S(t) + \mathbf{n}(t) + \mathbf{x}_I(t). \tag{9}$$

The conventional (forward-only) estimate of the covariance matrix defined as,

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} \tag{10}$$

where $E\{.\}$ represents the ensemble average; and $(.)^H$ is the Hermitian transposition operator. By applying temporal averaging over K observation snapshots taken from the signals incident on the sensor array, $\mathbf{R}$ can be approximated as, [22]

$$\mathbf{R} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k) \tag{11}$$

Substituting for $\mathbf{x}(t)$ from equation (9) in equation (11) yields,

$$\begin{aligned} \mathbf{R} \;=\; & \frac{1}{K}\sum_{t=1}^{K} \mathbf{A}_S \left[\mathbf{s}(k)\mathbf{s}(k)^H\right] \mathbf{A}_S^H + \mathbf{n}(k)\ \mathbf{n}(k)^H \\ & + \frac{1}{K}\sum_{t=1}^{K} \mathbf{A}_I \left[\mathbf{i}(k)\mathbf{i}(k)^H\right] \mathbf{A}_I^H. \end{aligned} \tag{12}$$

Finally, equation (12) can be written in compact form as,

$$\mathbf{R} = \mathbf{A}_S \mathbf{R}_{ss} \mathbf{A}_M^H + \sigma_n^2 \mathbf{I} + \mathbf{A}_I \mathbf{R}_{ii} \mathbf{A}_I^H \tag{13}$$

where $\mathbf{R}_{ss} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is an $M \times M$ desired users source waveform covariance matrix, $\mathbf{R}_{ii} = E\{\mathbf{i}(t)\mathbf{i}^H(t)\}$ is an $I \times I$ undesired users source waveform covariance matrix, σ_n^2 is the noise variance, and $\mathbf{I}$ is an identity matrix of size $N \times N$. In general the array correlation matrix obtained in equation (13) is referred as the covariance matrix only when the mean values of the signals and noise are zero. The arriving signals mean value must be necessarily zero because antennas can not receive d.c. signals.

III. ADAPTIVE BEAMFORMING ALGORITHM

An adaptive beamformer, which is shown in Fig. 2, consists of multiple antennas; complex weights, the function of which is to amplify (or attenuate) and delay the signals from each antenna element; and a summer to add all of the processed signals, in order to tune out the signals not of interest, while enhancing the signal of interest. Hence, beamforming is sometimes referred to as spatial filtering, since some incoming signals from certain spatial directions are filtered out, while others are amplified. The output response of the uniform linear array is given by,

$$y(k) = \mathbf{w}^H \mathbf{x}(k), \tag{14}$$

where $\mathbf{w}$ is the complex weights vector and $\mathbf{x}$ is the received signal vector given in equation (9). If $d(k)$ denotes the sequence of reference or training symbols known a priori at the receiver at time n, an error, $e(k)$ is formed as,

$$e(k) = d(k) - \mathbf{w}(k)^H \mathbf{x}(k). \tag{15}$$

This error signal e is used by the beamformer to adaptively adjust the complex weights vector $\mathbf{w}$ so that the mean-squared error (MSE) is minimized. It is intuitively reasonable that successive corrections to the weights vector in the direction of the negative of the gradient of the MSE function should eventually lead to minimum mean square error, at which point the weights vector assumes its optimum value. Recursive estimates for the unknown weight vector can be obtained adaptively via, [23]

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \mathbf{x}^*(k) f_e(k) \tag{16}$$

where $\mathbf{w}(k{+}1)$ denotes the weights vector to be computed at iteration $n+1$, μ is the algorithm step size, and $f_e(k)$ is a scalar function of the estimation error $e(k)$. The step size μ is related to the rate of convergence: in other words, how fast the algorithm reaches steady state. The smaller the step size the longer it takes the algorithm to converge. This means that a longer reference or training sequence is needed, which would reduce the payload and, hence, the bandwidth available for transmitting data.

The most popular variant of equation (16) is the least mean squares (LMS) algorithm for which the cost function to be minimized is given by,

$$J_2(k) = E\{e^2(k)\} \tag{17}$$

where $E\{.\}$ which results in an estimation error function $f_e^{LMS}(k)$ given as,

$$f_e^{LMS}(k) = e(k). \tag{18}$$

Hence, the weights update equation (16) for LMS becomes,

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\mathbf{x}^*(k)e(k). \qquad (19)$$

The cost function that is minimized for the least mean fourth (LMF) algorithm is given by,

$$J_4(k) = \frac{1}{4}E\{e^4(k)\} \qquad (20)$$

which results in an estimation error function $f_e^{LMF}(k)$ given by,

$$f_e^{LMF}(k) = e^3(k) \qquad (21)$$

In this case, the weights update equation (16) for LMF becomes,

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\mathbf{x}^*(k)e^3(k). \qquad (22)$$

Among other variants is the least mean mixed norm (LMMN) algorithm for which the cost function to be minimized is a linear mixture of the cost functions $J_2(k)$ and $J_4(k)$ for the LMS and LMF algorithms, respectively. It is given by,

$$J(k) = \frac{\delta}{2}J_2(k) + \frac{1-\delta}{4}J_4(k) \qquad (23)$$

where the parameter δ is called the norm mixing parameter such that $\delta \in [0, 1]$. This results in an estimation error function given by,

$$f_e^{LMMN}(k) = \delta e(k) + (1-\delta)e^3(k). \qquad (24)$$

The weights update equation (16) for LMMN becomes,

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\mathbf{x}^*(k)\left[\delta e(k) + (1-\delta)e^3(k)\right]. \qquad (25)$$

It is to be noted from equation (25) that the LMMN algorithm requires at each iteration only two more multiplications and one more addition than the LMS algorithm. Moreover, when $\delta = 1$ equation (25) becomes the weights update equation for the LMS algorithm. On the other hand, when $\delta = 0$ equation (25) becomes the wights update equation for the LMF algorithm. Judicious choice of δ thereby provides an algorithm with intermediate performance between that of the LMS and LMF, and a mechanism to mitigate the problem of instability within the LMF algorithm. Moreover, for operation in a statistically non-stationary environment, the mixing parameter δ may be adapted to match appropriately the properties of measured signals.

In order to ensure the stability and convergence of the LMMN algorithm, the adaptive step size parameter μ should be chosen within the range specified as, [13]

$$0 < \mu < \frac{1}{N.E\{x^2(k)\}}\left[\delta + (1-\delta)\frac{1}{6.E\{n^2(k)\}}\right] \qquad (26)$$

where $E\{x^2(k)\}$ is the input signal power and $E\{n^2(k)\}$ is the noise power. Analysis of the effect of varying the adaptive step size parameter μ in [2, 21] showed that μ should be chosen to be small in order to ensure numerical stability of the algorithm. Hence, in all the simulation results presented in Section 5 to follow have obtained with an adaptive step size value $\mu = 1\times10^{-3}$.

It is known that the LMF algorithm has better steady state performance than that of the LMS algorithm for applications in which the noise has a probability density function with short tail. However, its stability properties are worse than those of the LMS algorithm. On the other hand, the LMMN algorithm has better steady state performance than the LMS algorithm and better stability properties than the LMF algorithm [14]. It is for those reasons that we consider the application of LMMN algorithm to adaptive beamforming for robust interference suppression. The steady-state performance of the LMMN algorithm is a function of the norm mixing parameter δ. The steady-state performance is quantified in terms of the misadjustment, which is defined as,

$$M = \frac{1}{\sigma_n^2}\lim_{k\to\infty} E\{\mathbf{w}(k)^H\mathbf{x}(k)\} \qquad (27)$$

The effect of varying the norm mixing parameter δ on the misadjustment M is studied in Section 5. The purpose there is to derive an optimal value for δ for which the misadjustment M reaches a minimum value.

The weight initialization is arbitrary in the LMMN algorithm which makes it take longer (i.e., requires more iterations) to converge. To overcome this problem, we use the sample matrix inversion (SMI) algorithm to initialize the weights vector in the LMMN update equation (25). We further improve the performance of the LMMN algorithm by evaluating the initial weights vector in the LMMN weights update equation (25). SMI method is a block-data adaptive algorithm and is known to be the fastest algorithm for estimating the optimum weight vector. Because of its high complexity, SMI algorithm will be used only to estimate the initial weights vector $\mathbf{w}(0)$ which is obtained as,

$$\mathbf{w}(0) = \mathbf{R}^{-1}(0)\mathbf{r}(0) \qquad (28)$$

where the estimates of the covariance matrix $\mathbf{R}(0)$ and cross-correlation vector $\mathbf{r}(0)$ are given by,

$$\mathbf{R}(0) = \sum_{k=1}^{B}\mathbf{x}(k)\mathbf{x}^H(k) \qquad (29)$$

$$\mathbf{r}(0) = \sum_{k=1}^{B}\mathbf{x}(k)d^*(k). \qquad (30)$$

In equations (29) and (30), B represents the block size and is taken to be small just to ensure that the effect due to the change in the signal environment during the block acquisition does not affect the performance of the SMI algorithm. Also, a large block results in more matrix inversions making the algorithm computationally intensive.

The weight initialization as given in equation (28) is not any arbitrary value but an estimate of the optimum value computed by the SMI algorithm. This means that before the LMMN adaptation begins the antenna beam is already steered to an approximate direction of the desired signal, depending on the initial SMI weight estimate. In this way, the LMMN algorithm takes little time to converge. Also, after an estimate of the initial weights is made using the SMI algorithm, the SMI-initialized LMMN algorithm uses a continuous approach to adapt itself to the changing signal environment by updating the weights for every incoming sample. Since the initial convergence is faster, the SMI-initialized LMMN algorithm takes much less time than the LMMN algorithm to adapt to the signal environment changes. Therefore, the SMI-initialized LMMN algorithm is better suited for continuous transmission systems. Numerical examples in Section 7 illustrate the improved performance of the combined LMMN/SMI algorithm in comparison with the LMMN algorithm.

IV. SIMULATION RESULTS

Both the desired and interfering signals take the form of a simple complex sinusoidal-phase modulated signal. By doing so it can be shown in the simulations how interfering signals of the same frequency as the desired signal can be separated to achieve rejection of co-channel interference. For simplicity purpose the reference signal $d(k)$ is considered to be the same as the desired signal.

A. Optimal Value of Norm Mixing Parameter (δ)

To find the optimal value of the norm mixing parameter δ, Monte Carlo simulations are carried to plot the misadjustment M as defined in equation (27) versus δ. The mixing parameter δ is used to calculate the tap weights according to the LMMN weight update equation (25). The LMMN step size is chosen as $\mu = 1 \times 10^{-3}$. The mixing parameter δ is chosen as 10 equispaced points in $[0, 1]$. The input signal, $\mathbf{x}(k)$ is zero-mean and uniformly distributed with unity power. The noise signal is also zero-mean and it is obtained by adding a Gaussian distributed noise of power $\sigma^2_{n_1} = 0.1$ and a uniformly distributed noise of power $\sigma^2_{n_2} = 1.0$. The signal-to-noise ration (SNR) is 10 dB. The values of misadjustment M are computed at the steady state, after 10^5 iterations, from equation (27) by averaging over 50 Monte Carlo trials. The variation of the misadjustment M with respect to δ is shown in Fig. 3, where it is clear that the misadjustment curve has a well defined minimum at $\delta = 0.40$. Therefore by choosing $\delta = 0.40$, it is expected that the LMMN algorithm performs better than both LMS ($\delta = 0$) and LMF ($\delta = 1$) algorithms. Hence, all simulation results for the LMMN algorithm presented in Section 5 to follow are carried out with a mixing parameter value $\delta = 0.40$ in order to ensure optimum steady state performance.

B. Beampattern

Consider an array of four elements ($N = 4$) and half-wavelength spacing ($d = 0.5\lambda$). The array is to

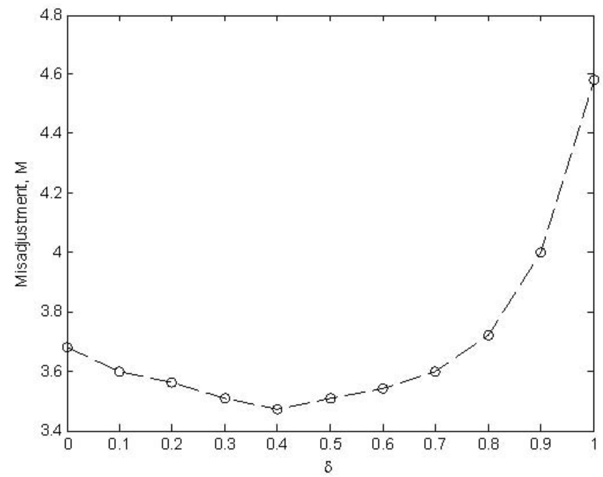

Fig. 3. Misadjustment M vs. LMMN norm mixing parametr δ (SNR= 10 dB, $\mu = 1 \times 10^{-3}$).

maximize output radiation towards a source signal arriving at an angle $\theta_S = 0^o$ having a signal-to-noise ratio $SNR = 10$dB. The array is also designed to mitigate an interference signal arriving at an angle $\theta_I = -60^o$ having a signal-to-interference ratio $SIR = -10$dB. The number of iterations is 6000 for both algorithms. The step size for both LMS and LMMN algorithms is taken as $\mu = 1 \times 10^{-3}$ whereas the norm mixing parameter δ is fixed at its optimum value obtained in Section 4 as $\delta = 0.40$. Results are presented in Fig. 4 where the solid line represents the beampattern obtained using the LMS algorithm and the dashed line represents the beampattern obtained from the proposed LMMN algorithm with SMI initialization. It is evident that the pattern nulls in the case of the LMS algorithm (solid line in Fig. 4) are not deep enough to cancel the effect of interfering signals. This means that the LMS algorithm did not converge to the optimum weights solution within the given number of iterations. On the other hand, the SMI-initialized LMMN algorithm (dashed line in Fig. 4) is capable of generating deep pattern nulls (90dB below the maximum) which are strong enough to cancel the effect of the interfering signals. This means that the SMI-initialized LMMN algorithm converges faster as it reached to the optimum weights solution within the given number of iterations. This is due to the fact that the initialization of the weights vector in equation (25) was obtained from the SMI algorithm as described in equations (28) to (30).

C. Convergence

Here, simulations are carried out for an array with four elements ($N = 4$) and half-wavelength spacing ($d = 0.5\lambda$). The array is to maximize output radiation towards a source signal arriving at an angle $\theta_S = 0^o$ having a signal-to-noise ratio $SNR = 10$dB. The array is also designed to mitigate an interference signal arriving at an angle $\theta_I = -60^o$ having a signal-to-interference ratio

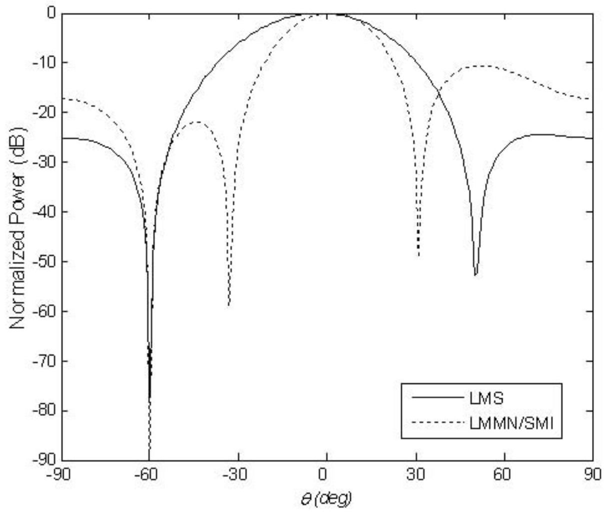

Fig. 4. Output beampattern using LMS and proposed LMMN/SMI algorithm ($N = 4$, $d = 0.5\lambda$, $SNR = 10$ dB, $SIR = -10$dB, $\mu = 1 \times 10^{-3}$, $\delta = 0.40$, $\theta_S = 0^o$, $\theta_I = -60^o$).

$SIR = -10$dB. The step size for both LMS and LMMN algorithms is taken as $\mu = 1 \times 10^{-3}$ whereas the norm mixing parameter δ is fixed at its optimum value obtained in Section 4 as $\delta = 0.40$. The convergence of the array weights is studied by plotting in Fig. 5 the magnitude of the array weights vector $\mathbf{w}(1)$ versus number of iterations for both LMS algorithm and proposed LMMN algorithm with SMI initialization, respectively. It is evident from that the array weights obtained using the LMS algorithm (solid line in Fig. 5) did not converge to the steady state optimum solution within the given number of iterations. On the other hand when the proposed SMI-initialized LMMN algorithm (dashed line in Fig. 5) is used, the array weights converge to the stable value within the given number of iterations since the initialization of the weights vector in the LMMN update equation (25) was done using the SMI algorithm as described in equations (28) to (30). This verifies the improved convergence rate that is achieved when the proposed SMI-initialized LMMN algorithm is used.

D. Mean Square Error (MSE)

The convergence of the beamforming algorithm is examined by plotting in Figs. 6 and 7 the Mean Square Error (MSE) versus number of iterations for both the LMS algorithm and proposed SMI-initialized LMMN algorithm, respectively. Results of Fig. 7 show a significant improvement in terms of a reduced MSE when the SMI-initialized LMMN algorithm is used indicating that it has a faster convergence rate when compared to the LMS algorithm of Fig. 6. This verifies the improved accuracy that is obtained when the proposed SMI-initialized algorithm is used.

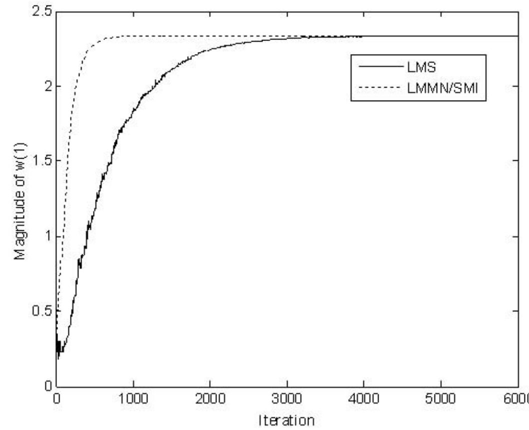

Fig. 5. Convergence of $\mathbf{w}(1)$ using both LMS and SMI-initialized LMMN algorithm ($N = 4$, $d = 0.5\lambda$, $SNR = 10$ dB, $SIR = -10$dB, $\mu = 1\times 10^{-3}$, $\delta = 0.40$, $\theta_S = 0^o$, $\theta_I = -60^o$).

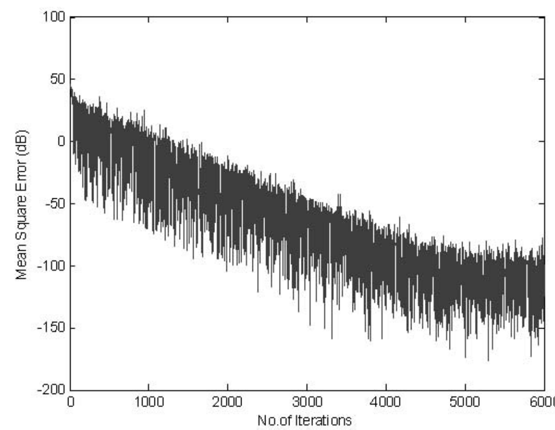

Fig. 6. Mean square error vs. number of iterations for LMS algorithm ($N = 4$, $d = 0.5\lambda$, $SNR = 10$ dB, $SIR = -10$dB, $\mu = 1 \times 10^{-3}$, $\theta_S = 20^o$, $\theta_I = -40^o$).

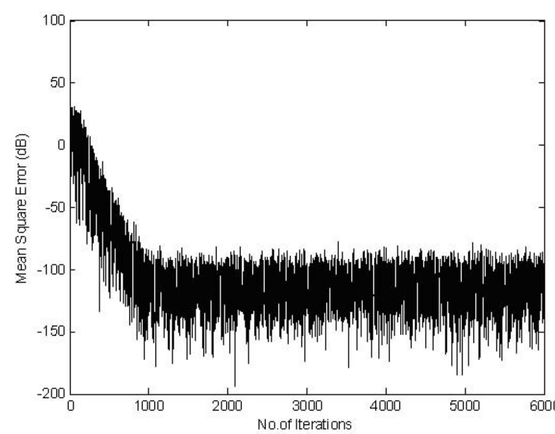

Fig. 7. Mean square error vs. number of iterations for SMI-initialized LMMN algorithm($N = 4$, $d = 0.5\lambda$, $SNR = 10$ dB, $SIR = -10$ dB, $\mu = 1 \times 10^{-3}$, $\delta = 0.40$, $\theta_S = 20^o$, $\theta_I = -40^o$).

V. CONCLUSIONS

An accurate and computationally-efficient adaptive beamforming algorithm based on LMMN with SMI initialization was presented. The algorithm uses a mixing parameter δ which controls the proportions of the error norms and offers an extra degree of freedom within the adaptation. Monte Carlo simulations show that the misadjustment curve has a minimum at $\delta = 0.40$ which means that the LMMN algorithm has an optimum steady-state performance at this mixing parameter value. The convergence of the algorithm is further improved by employing SMI to initialize the weights vector in the LMMN update equation. Hence, the SMI-initialized LMMN algorithm provides better steady state performance when compared to the least mean squares (LMS) algorithm and better stability properties when compared to the least mean fourth (LMF) algorithm. Simulation results obtained show that the proposed SMI-initialized LMMN algorithm performs better when compared to the other algorithms. The improved performance of the proposed SMI-initialized LMMN algorithm takes the form of faster convergence rate, less mean square error, as well as deeper nulls placed accurately in the directions of interference signals. These features make the proposed algorithm suitable for the design and implementation of practical smart antenna systems.

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