

# A General Framework for Mixed Structured/Unstructured PEEC Modelling

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**Abstract** – The paper proposes a re-formulation of PEEC modeling in terms of dual discretization of surface conductors. The result is a precise formalization of the PEEC tessellation and gives the possibility of handling general meshes, both structured and unstructured. This fact allows the use of triangular meshes only where necessary, leaving orthogonal discretization whenever possible and improving computational performances. The method, referred to as dual-PEEC, is validated by solving a structure referenced in literature and finally applied to the computation of the input impedance of a spiral inductor.

**Keywords:** PEEC, unstructured mesh, and dual discretization.

## I. INTRODUCTION

Integral approaches to Maxwells equations for the modeling of interconnects and packaging structures are usually preferred to differential ones, because they solve the field equation in terms of the sources of fields, located on the metallic structures. This allows to discretize the conducting part of the domain only, accounting rigorously for the regularity conditions of fields at infinity.

Among other integral techniques, the Partial Element Equivalent Circuit (PEEC) leads to the description of the electromagnetic problems in terms of circuit parameters which describe the magnetic and electric coupling between currents and charges of the discretized structure. The main advantages of PEEC models are the possibility of integrating lumped parameters into the electromagnetic structure and the availability of both time and frequency analysis in standard SPICE-like solvers.

The classic PEEC approach is restricted to orthogonal discretization of the structures under study. Recently the scheme has been extended to non-orthogonal [1, 2] and triangular [3, 4] meshes, but the proposed techniques lack of a systematic background for their assumptions.

The aim of this work is to generalize the PEEC methodology by the use of the topological concept of *duality*. The result is the exact formalization of the PEEC tessellation gaining in addition the possibility of handling general meshes, both structured and unstructured. This fact allows the use of triangular meshes only where necessary (i.e., curved or slanting electrodes), leaving

orthogonal discretization in regular regions, with a global improvement of computational performances.

The paper is structured as follows: in Section II the original PEEC scheme is briefly described; Section III gives a general overview of dual discretization which is used as a framework of the mathematical formulation of Section IV. Two examples are investigated in Section V and finally Section VI draws some conclusions.

## II. ORIGINAL PEEC SCHEME

The first systematic description of the Partial Element Equivalent Circuit, PEEC, goes back to the middle 70s, in a work by A.E. Ruehli [5]. The original PEEC formulation is based on the circuit interpretation of the terms of the Mixed Potential Integral Equation (MPIE),

$$\frac{\vec{J}(\vec{r}, t)}{\sigma} + \frac{d\vec{A}(\vec{r}, t)}{dt} + \nabla\varphi(\vec{r}, t) = 0 \quad (1)$$

$\vec{A}$  and  $\varphi$  are the retarded magnetic vector and electric scalar potential, respectively,

$$\vec{A}(\vec{r}, t) = \frac{\mu}{4\pi} \int_{\Omega} \frac{\vec{J}(\vec{r}', t')}{|\vec{r} - \vec{r}'|} d\Omega \quad (2)$$

$$\varphi(\vec{r}, t) = \frac{1}{4\pi\epsilon} \int_{\Omega} \frac{\rho(\vec{r}', t')}{|\vec{r} - \vec{r}'|} d\Omega, \quad (3)$$

where  $t'$  is the retardation time,

$$t' = t - \frac{|\vec{r} - \vec{r}'|}{c}.$$

The solution of the electromagnetic problem is achieved by discretizing equation (1) on the system of surface cells shown in Fig. 1(a). Current and charge densities are expanded in terms of piecewise constant pulse functions. The same functions are also used as test functions in a Galerkin weighted residual scheme [6].

- Partial self and mutual inductances between nodes, representing the magnetic field couplings;
- Resistive terms between nodes, representing the Joule losses in the conductive material;
- Coefficients of potential at each node, representing the electric field couplings.

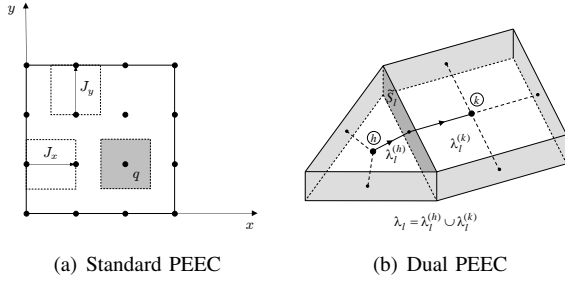


Fig. 1. (a) Standard 2D discretization of current density (dashed cells) and charge density (gray cell), (b) dual discretization: the couple *primal edge* (straight line)/*dual face* (dark gray) correspond to a two terminal circuit component.

### III. DUAL DISCRETIZATION

The work of E. Tonti has shown how the formulation of the electromagnetic field can be expressed in terms of dual relations and how this duality can be exploited in its numerical solution [7]. The conceptual framework of the finite formulation of electromagnetic fields is based on the definition of two sets of global variables: *source* and *configuration*. In the present application *source variables* are electric charge and electric current. *Configuration variables* are voltage, electric potential, magnetic vector potential. The product of a source variable by a configuration variable gives an *energy variable*. Physical variables are associated to spatial elements. In [8] it is shown that configuration variables are associated with space elements endowed with inner orientation (i.e., the orientation of the space element lies on the element itself); on the contrary source variables are associated with space elements endowed with outer orientation (i.e., the orientation of the space element depends on the space in which the element is embedded). The complex of cell with inner orientation is called *primal complex*, whereas the *dual complex* is endowed with outer orientation. Primal and dual complexes of cells are interlinked: primal nodes correspond to dual volumes, primal edges to dual faces, primal faces to dual edges and primal volumes to dual nodes. Figure 2 shows the generation of primal and dual complex and their spatial duality, assuming primal complex as simplicial. It is important to note that it is possible to assume the dual complex as simplicial, when source variables are chosen as unknowns (in this work the latter representation is adopted).

Table 1 reports the spatial assignment of the variables used in PEEC modeling: scalar potentials are assigned to primal nodes, voltage drops to primal edges, currents to dual faces and charges to dual volumes. This definition of duality relations of grids and the rigorous assignment of physical variables to spatial entities allows the possibility of giving a general framework of PEEC modeling, i.e., allowing the use of unstructured and mixed structured/unstructured complexes of cells. Besides the

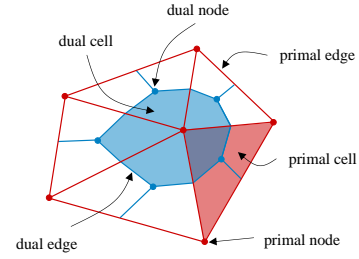


Fig. 2. 2D primal and dual complex.

generality of dual discretization, in this paper only surface discretization is investigated, leaving the generalization to volume discretization to a further work.

Making reference to Fig. 1(b) the discretization of conductors is made by prisms with triangular or rectangular basis. This tessellation constitutes the *dual complex* of cells. The *primal complex* is obtained by connecting the centroids of dual volumes by piecewise segments crossing the lateral faces of prisms.

Table 1. Variable assignment to spatial elements.

Type	Variable	Spatial Element
source	current, $i$	dual face, $\tilde{\Sigma}$
	charge, $q$	dual volume, $\tilde{\Omega}$
configuration	voltage, $u$	primal edge, $\lambda$
	potential, $\varphi$	primal node $P$

### IV. MATHEMATICAL FORMULATION

#### A. Basis Functions

When using a general approach to PEEC models, a local interpolation inside dual volumes linking the current density  $\vec{J}$  in equation (1) to the global variable  $i$  through dual faces is needed. By referring to the generic dual volume or cell  $k$  identified by a prism of thickness  $\delta_k$ , and assuming a uniform  $\vec{J}$  distribution over the electrode thickness, *facet* elements basis functions  $\vec{w}$  are chosen,

$$\vec{J}_k = \frac{1}{\delta_k} \sum_{m=1}^{N_F} i_{km} \vec{w}_{km} \quad (4)$$

where  $N_F$  is the number of dual faces in each cell (3 or 4 in our case),  $i_{km}$  is the current through the  $m$ th dual face of cell  $k$ . Many choices for  $\vec{w}$  are possible but div-conforming basis functions must be selected for edges belonging to triangles, rectangles or common between them [9].

With reference to a local frame  $(u, v, w)$  (Fig. 3(a)) the following facet functions for triangles are selected [10],

$$\begin{aligned} \vec{w}_k^3(u, v) = & N_j(u, v) \nabla N_i(u, v) \times \vec{n} \\ & - N_i(u, v) \nabla N_j(u, v) \times \vec{n} \end{aligned}$$

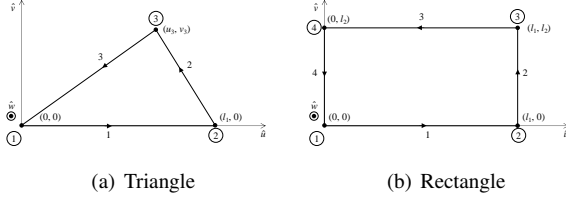


Fig. 3. Plane triangle and rectangle in local frame  $(u, v, w)$ .

where the functions  $N_i(u, v)$  and  $N_j(u, v)$  are the standard nodal functions related to the extreme nodes of edge  $k$  and  $\vec{n} = \hat{w}$  is the unit vector orthogonal to the triangle surface. The analytical expression of these shape functions is,

$$\begin{aligned}\vec{w}_1^3(u, v) &= \frac{u - u_3}{l_1 v_3} \hat{u} + \frac{v - v_3}{l_1 v_3} \hat{v} \\ \vec{w}_2^3(u, v) &= \frac{u}{l_1 v_3} \hat{u} + \frac{v}{l_1 v_3} \hat{v}, \\ \vec{w}_3^3(u, v) &= \frac{u - l_1}{l_1 v_3} \hat{u} + \frac{v}{l_1 v_3} \hat{v}.\end{aligned}$$

Figure 4(a) shows the vector plot of  $\vec{w}_1^3$  for a particular triangle in the  $x, y$  plane. For rectangular cells, special basis functions are built with the same properties of the ones for triangles.  $\vec{w}_k^4$  has only the component orthogonal to edge  $k$  and its magnitude linearly decrease from edge  $k$  to the opposite one. The analytic expression in the local  $(u, v, w)$  coordinate system is,

$$\begin{aligned}\vec{w}_1^4(u, v) &= \frac{v - l_2}{l_1 l_2} \hat{v} \\ \vec{w}_2^4(u, v) &= \frac{u}{l_1 l_2} \hat{u}, \\ \vec{w}_3^4(u, v) &= \frac{v}{l_1 l_2} \hat{v}, \\ \vec{w}_4^4(u, v) &= \frac{u - l_1}{l_1 l_2} \hat{u}.\end{aligned}$$

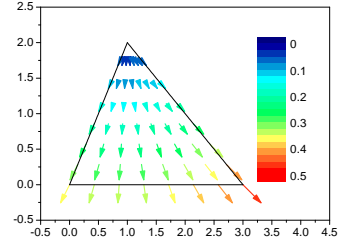
An example of a vector plot is reported in Fig. 4(b). It is easy to prove the continuity of the normal component of  $\vec{w}$  also for common edges shared by rectangles and triangles.

These basis functions have the property of being affine with respect to the  $u, v$  coordinates. This fact allows the possibility of using analytical formulas to solve the surface integrals described in the Section IV(b).

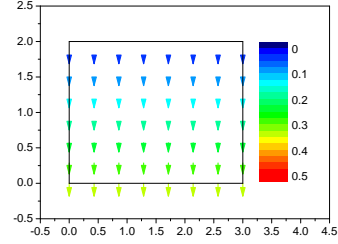
The surface charge density is expanded in terms of piecewise constant functions on each triangular/rectangular cell.

### B. Partial Element Calculation

The parameter extraction can be done by following the same rationale of the original PEEC formulation, i.e., testing MPIE equation (1) with the same basis function used to expand current and charge densities. An equivalent approach is here proposed, by making reference to energetic considerations.



(a) Triangle of vertices  $(0, 0), (3, 0), (1, 2)$



(b) Rectangle of vertices  $(0, 0), (3, 0), (3, 2), (0, 2)$

Fig. 4. Vector plot of facet element  $\vec{w}_1$  over (a) triangles and (b) rectangles.

1. *Extraction of resistances:* The Joule's power losses in the  $k$ th cell are,

$$\begin{aligned}P_k &= \int_{\Omega_k} \frac{J_k^2}{\sigma} d\Omega_k \\ &= \frac{1}{\delta_k^2 \sigma} \int_{\Omega_k} \left( \sum_{n=1}^{N_F} i_{kn} \vec{w}_{kn} \right) \cdot \left( \sum_{m=1}^{N_F} i_{km} \vec{w}_{km} \right) d\Omega_k.\end{aligned}$$

When only surface approximations are taken into account, the volume integral can be rearranged into a surface one,

$$P_k = \frac{1}{\delta_k \sigma} \int_{\Sigma_k} \left( \sum_{n=1}^{N_F} i_{kn} \vec{w}_{kn} \right) \cdot \left( \sum_{m=1}^{N_F} i_{km} \vec{w}_{km} \right) d\Sigma_k.$$

The power related to the current flowing through the generic  $\alpha$ th dual face of cell  $k$  is,

$$P_{k\alpha} = i_{k\alpha} \sum_{m=1}^{N_F} \frac{i_{km}}{\delta_k \sigma} \int_{\Sigma_k} \vec{w}_{k\alpha} \cdot \vec{w}_{km} d\Sigma_k = i_{k\alpha} u_{k\alpha}$$

where

$$\begin{aligned}u_{k\alpha} &= \sum_{m=1}^{N_F} \frac{i_{km}}{\delta_k \sigma} \int_{\Sigma_k} \vec{w}_{k\alpha} \cdot \vec{w}_{km} d\Sigma_k \\ &= \sum_{m=1}^{N_F} R_{km} i_{km}\end{aligned}$$

$$R_{km} = \frac{1}{\delta_k \sigma} \int_{\Sigma_k} \vec{w}_{k\alpha} \cdot \vec{w}_{km} d\Sigma_k.$$

The voltage drop over half primal edge  $\alpha$  depends on currents through all dual faces of cell  $k$ .

2. *Extraction of partial inductances:* Resorting to magnetic vector potential defined in equation (2), the magnetic energy associated to  $k$ th cell is,

$$\begin{aligned} W_k &= \frac{1}{2} \int_{\Omega_k} \vec{J}_k \cdot \vec{A} d\Omega_k \\ &= \frac{1}{2} \frac{\mu_0}{4\pi} \int_{\Omega_k} \vec{J}_k \cdot \int_{\Omega_h} \sum_{h=1}^{N_{\bar{V}}} \frac{\vec{J}_h}{|\vec{r} - \vec{r}'|} d\Omega_h d\Omega_k \end{aligned}$$

where  $N_{\bar{V}}$  is the number of dual volumes. By using the expansion equation (4) and extracting only the magnetic coupling between the current  $\alpha$  of the  $k$ th cell, and current  $\beta$  of cell  $h$ , one has,

$$W_{k\alpha,h\beta} = \frac{1}{2} \frac{\mu_0}{4\pi} \int_{\Omega_k} \frac{i_{k\alpha} \vec{w}_{k\alpha}}{\delta_k} \cdot \int_{\Omega_h} \frac{i_{h\beta} \vec{w}_{h\beta}}{\delta_h |\vec{r} - \vec{r}'|} d\Omega_h d\Omega_k.$$

Finally, when considering surface complex of cells only, the partial inductance is,

$$\begin{aligned} W_{k\alpha,h\beta} &= \frac{1}{2} L_{k\alpha,h\beta} i_{k\alpha} i_{h\beta} \\ L_{k\alpha,h\beta} &= \frac{\mu_0}{4\pi} \int_{\Sigma_k} \vec{w}_{k\alpha} \cdot \int_{\Sigma_h} \frac{\vec{w}_{h\beta}}{|\vec{r} - \vec{r}'|} d\Sigma_h d\Sigma_k. \end{aligned} \quad (5)$$

3. *Extraction of coefficients of potential:* Making reference to equation (3), the electric energy related to the  $k$ th cell is,

$$\begin{aligned} W_k &= \frac{1}{2} \int_{\Omega_k} \rho_k \varphi d\Omega_k \\ &= \frac{1}{2} \frac{1}{4\pi\epsilon_0} \int_{\Omega_k} \rho_k \sum_{h=1}^{N_{\bar{V}}} \int_{\Omega_h} \frac{\rho_h}{|\vec{r} - \vec{r}'|} d\Omega_h d\Omega_k. \end{aligned}$$

With an approach similar to the one used for the extraction of partial inductances, it is possible to evaluate the electric energy due to the mutual coupling between the global charge  $q_k$  in cell  $k$  and  $q_h$  in cell  $h$ ,

$$W_{k,h} = \frac{1}{2} p_{k,h} q_k q_h$$

where

$$p_{k,h} = \frac{1}{4\pi\epsilon_0 \Sigma_k \Sigma_h} \int_{\Sigma_k} \int_{\Sigma_h} \frac{1}{|\vec{r} - \vec{r}'|} d\Sigma_h d\Sigma_k. \quad (6)$$

### C. Equivalent Circuit

Figure 5 shows the resulting two terminal component obtained by assembling all the previous contributions, and it is representative of the basic topological structure *primal edg dual face* presented in Fig. 1(b). The unstructured PEEC two terminal components can be assembled to obtain the MNA matrix to be solved by a general purpose SPICE-like network simulator. With respect to standard PEEC model, the use of unstructured meshes introduces a local resistive mutual coupling. The use of dependent current sources allows to deal with this problem without increasing significantly the complexity of the circuit.

Figure 6 outlines the sequence of necessary steps for the application of dual-PEEC methodology to the solution of a full Maxwell problem. It is worth noting that the

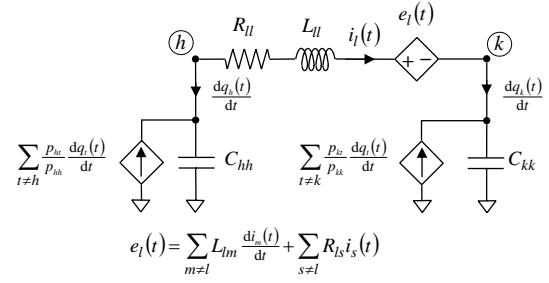


Fig. 5. Elementary branch of PEEC model.

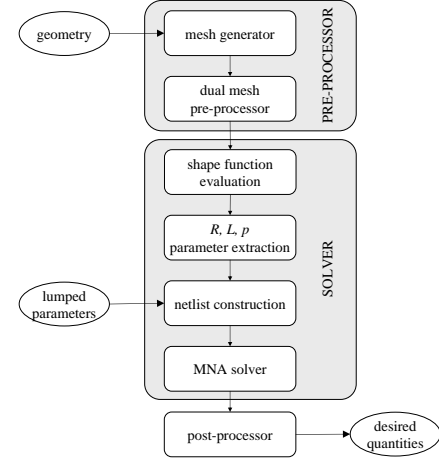


Fig. 6. Solver structure.

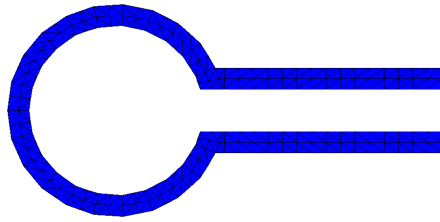
added complexity of the method due to the generation of the dual complex and the definition of basis function is negligible with respect to standard technique.

### D. Remarks

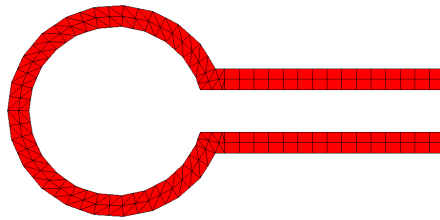
Equations (5) and (6) present a double surface integral, having the Green's function as kernel (in some cases multiplied by an affine combination of the  $(u, v)$  coordinates). To speed up the effort of the partial element computation, the inner integrals of equations (5) and (6) are solved by analytical formulas reported in [11, 12]. The outer integral is solved by using adaptive Gauss-Kronrod quadrature rule. The number of quadrature points is automatically chosen depending on the distance between cells: the larger the distance, the smaller the number of points needed to reach the convergence of the quadrature rule. Moreover, the computational time for the evaluation of inductance and coefficient of potential matrices can be reduced by considering that these matrices must be symmetric.

The standard PEEC formulation can be found as a particular case of the dual formulation when both charge and current densities are expanded in terms of piecewise constant pulse functions.

The resulting dual-PEEC circuit is similar to the standard PEEC model, with the exception of a local



(a) Triangular mesh



(b) Triangular/ quadrilateral mesh

Fig. 7. Loop antenna test case: (a) triangular discretization used as reference, (b) mixed triangular/quadrilateral discretization available with dual-PEEC modeling.

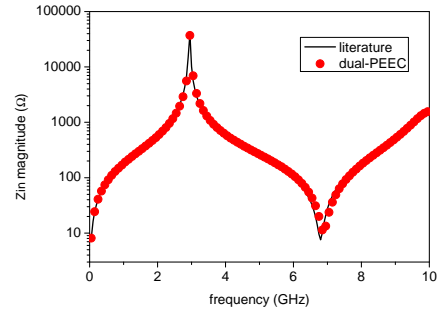
resistive coupling due to the use of unstructured meshes. The use of current controlled voltage sources allows to deal with this problem with a negligible computational effort (see Fig. 5) [13].

In frequency domain the inclusion of retardation is straightforward and makes PEEC equivalent to a full wave solution of Maxwell’s equations. It can be done by multiplying the non retarded mutual inductances and coefficients of potential by  $\exp(-j\omega\tau_{hk})$ , where  $\tau_{hk}$  is the delay time between the center coordinates of primal edges and dual volumes, respectively.

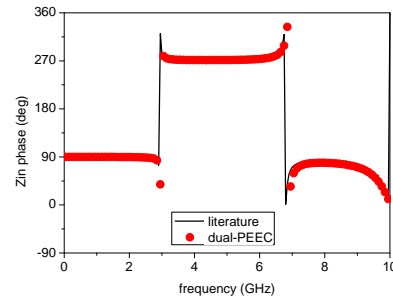
## V. EXAMPLES

### A. Model Validation

The dual-PEEC is first used to solve a benchmark loop antenna structure with known solution, already discussed in literature [14, 15]. Figure 7 shows the meshes adopted for the comparison: the structure of Fig. 7(a) is discretize by triangles and is solved by a FEM technique, whereas the mixed discretization of Fig. 7(b) is solved by the proposed procedure. Figure 8 compares the magnitude and phase of the input impedance evaluated in a full-wave regime. The results show a satisfactory agreement with respect to the literature ones.



(a) magnitude



(b) phase

Fig. 8. Comparison of input impedance evaluation: (a) magnitude, (b) phase.

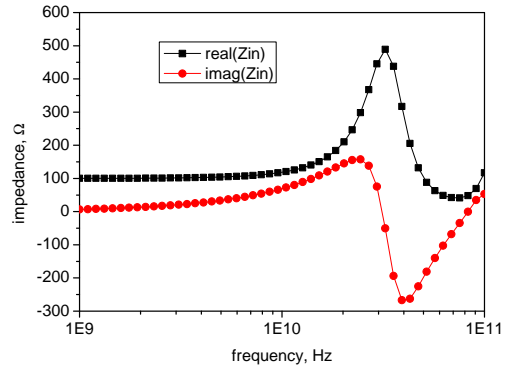


Fig. 9. Spiral inductor test case: input impedance.

### B. Spiral Inductor

The dual-PEEC technique is then used to solve the a surface metallic spiral inductor above a (finite) ground plane, ended by a lumped 100  $\Omega$  resistor. The input impedance (real and imaginary part) is reported in Fig. 9. Three different meshes are compared in Table 2 with respect to their geometrical parameters and computational time (in seconds on a Intel Centrino 1.5 GHz). The first mesh Fig. 10(a) is made by unstructured triangles (Mesh 1), the second one Fig. 10(b) with regular triangles (Mesh 2), the last one Fig. 10(c) with mixed triangles and

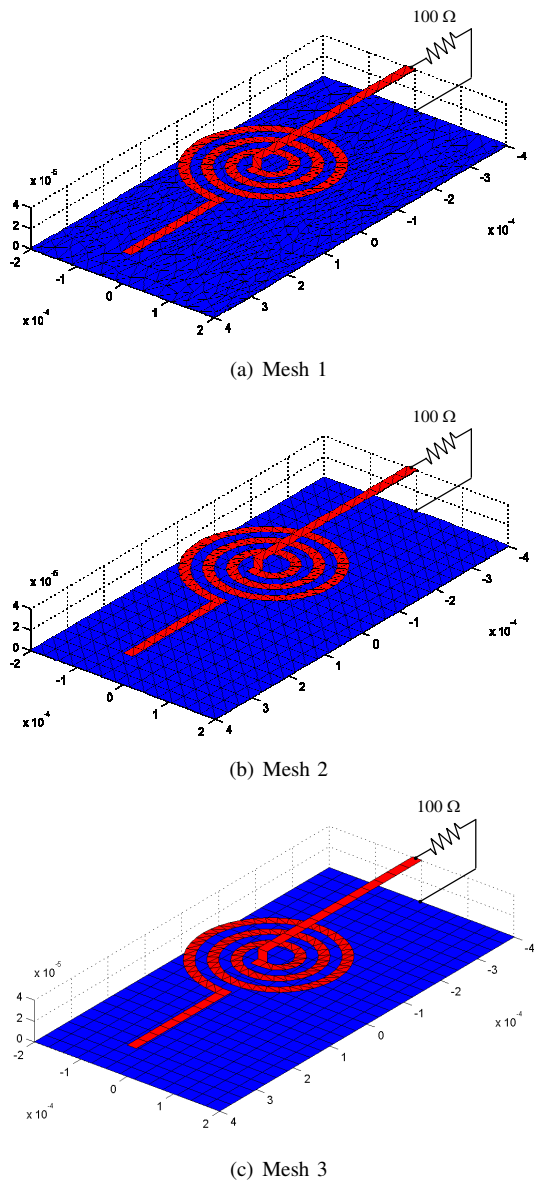


Fig. 10. Spiral inductor test case: (a) unstructured triangular mesh, (b) regular triangular mesh, (c) mixed triangular/rectangular mesh.

rectangles (Mesh 3). The three meshes share the same average edge length.

While no significant differences are observed in the values of impedance, the possibility of using together both structured and unstructured complexes of cells increases the computational efficiency of the method. In fact, given the maximum size of cell's edge (related to the shortest wavelength), the parameter extraction over rectangular dual is faster than over triangles and also the number of unknowns is lower, as shown in Table 2.

## VI. CONCLUSIONS

In this paper the generalization of PEEC modeling under the framework of dual discretization is presented.

Table 2. Comparison of complexity of three meshes in Fig. 10. In brackets are the numbers of non-floating edges. Computational times are in seconds. The solution time is intended for one frequency point.

		Mesh 1	Mesh 2	Mesh 3
Geometry	nodes	757	646	646
	faces	1222	900	539
	edges	1977 (1689)	1502 (1198)	1141 (837)
Time	inductance	2995	768	189
	coeff. pot.	884	219	68
	solution	343	127	37
	total time	4222	1114	294

Relations of duality are exploited for the exact assignment of electromagnetic quantities to spatial elements. In this way it is possible to extend standard PEEC to unstructured and mixed structured/unstructured complexes of cells. The proposed technique, named dual-PEEC has proved to be computationally efficient because triangular cells are used only where necessary (e.g. for curved or slanting conductors). The additional computational effort due to mutual resistances is negligible and can be easily handled by adding a proper term in the current controlled voltage source. Finally, dual-PEEC formulation can be improved by using the same “tricks” already tested for standard PEEC (fast multipole method, wavelet transform, model order reduction, ...).

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