Quasi Monte Carlo Integration Technique for Method of Moments Solution of EFIE in Radiation Problems

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Abstract – In this work, a Quasi Monte Carlo Integration (QMCI) Technique using Halton Sequence is proposed for the Method of Moments (MoM) solution of the Electric Field Integral Equation (EFIE) in radiation problem. It is found that this scheme is capable of handling the singularity issue in the EFIE automatically and at the same time provides solution to the radiation problems very efficiently.

I. INTRODUCTION

Multidimensional numerical quadratures are of great importance in many practical areas, ranging from radiation/scattering problems in computational electromagnetics to atomic physics. The EFIE in solution of MoM for scattering problems involves multidimensional integrals especially when the Galerkin's technique for solution is employed. It is well known that a *D* dimensional scattering problem using Galerkin's technique involves solution of 2*D* dimensional integral equation. Gaussian quadrature methods, on the one hand, yield precise results with relatively few integrand evaluations, but they are not too robust and work best for very smooth functions and the time complexity in numerical quadrature methods increases as the dimension of the problem increases. Monte Carlo methods [1-3], on the other hand, impose few requirements on the integrand, but are known to converge slowly. It is an integration approach that is well suited for irregular or singular integrands and requires no analytic knowledge about the form of the integrand. The conventional Monte Carlo integration (MCI) method is independent of the dimension of the integral, and that is why MCI is the only practical method for many high-dimensional problems.

QMCI methods are based on the idea that random Monte Carlo techniques can often be improved by replacing the underlying source of random numbers with a more uniformly distributed deterministic sequence. The fundamental feature underlying all QMCIs, however, is the use of a quasi-random number (QRN) sequences in place of the usual pseudorandom numbers which often improves the convergence of the numerical integration.

One of the key issues in the solution of the EFIE using Galerkin's technique is the singularity appearing the Green's function kernel of the Integral Equation. The type of the singularity is weak in nature. Several techniques [4-7] have been used in the past to deal with the issue of singularity in order to solve the problem. The conventional MCI takes care of the singularity aspect without employing any analytical techniques such as singularity subtraction/removal, polar co-ordinate transformation, etc. and implements the idea just by avoiding the random points to fall in the singular region. This happens by including a simple statement in the program code used for the simulation purpose. However, the proposed Halton sequence in QMCI takes care of the singularity issue automatically without even modification or inclusion of any condition in the program code and provides solution to the problem much faster than the conventional MCI with randomly generated point sequences. The inherent nature of the Halton sequences automatically avoids inclusion of singular points in the integration.

II. MATHEMATICAL CONCEPT

The idea of Monte Carlo integration is to evaluate an integral using random sampling of points for function evaluation. In this method, if I is a D dimensional integral,

$$I = \int_{\Omega} f(x_1, x_2, ..., x_D) dx_1 dx_2 ... dx_D \qquad (1)$$

where $f(x_1, x_2, ..., x_D)$ is the integrand function and Ω is domain of integration in D dimensional space. The Monte Carlo integration is done by independently sampling N random points $\{(x_{11}, x_{12}, ..., x_{1D}), (x_{21}, x_{22}, ..., x_{2D}), ..., (x_{N1}, x_{N2}, ... x_{ND})\}$ in Ω, according to some convenient probability density function $p(x_{i1}, x_{i2}, ..., x_{iD})$, and then computing the estimate,

$$F_N = \frac{\Omega}{N} \sum_{i=1}^{N} \frac{f(x_{i1}, x_{i2}, ..., x_{iD})}{p(x_{i1}, x_{i2}, ..., x_{iD})}. \quad (2)$$

Here the notation F_N is used rather than I to emphasize that the result is approximate, and that its properties depend on how many sample points are chosen. If $p(x_{i1}, x_{i2}, ..., x_{iD})$ is the uniform probability density, then the integral is simply,

$$I \cong \frac{\Omega}{N} \sum_{i=1}^{N} f(x_{1i}, x_{2i}, ..., x_{Di}). \quad (3)$$

Unlike the conventional Monte Carlo integration that uses sampling of random points in Ω, the Quasi Monte Carlo Integration methods use the sampling points with uniform probability distribution that are more evenly distributed than the random points over the domain Ω. The classical QMCI method replaces the independent random points used in MCI by a deterministic set of distinct points. The problem of clustering of random numbers in the domain can be removed by the use of Halton numbers. The use of quasi random sequences in place of the usual pseudorandom numbers often improves the convergence of the numerical integration.

There are several well-known constructions for QRN sequences. In the one dimensional case, it is achieved, for example, by the Van der Corput sequence [3]. This construction uses a prime number as base for generation of numbers between 0 and 1, obtained by reversing the digits in the representation of some sequence of integers in a given base. To obtain a QRN sequence in several dimensions, we use a different radical inverse sequence in each dimension.

The classic example of this construction in several dimensions is the Halton sequence [8]. In one dimension for a prime base p_n, the n^{th} number in the sequence corresponding to the digit n is obtained by the following steps [1].

For each n:

1. n is written as a number in base p_n. Thus if p_n = 3 and n = 22, then 22 in base 3 is written as 22 = $2 * 3^2 + 1 * 3^1 + 1 * 3^0$ = 211.
2. The digits are reversed and a radix point (i.e., a decimal point base p_n) is put in front of the sequence (in the example, we get 0.112 base 3).
3. The sequence for one dimension is obtained by application of the above process for different values of n.

Every time the number of digits in n increases by one place, n's digit-reserved fraction becomes a factor of p finer-meshed. So, at each step as n increases points of Halton sequence are better and better filling Cartesian grids. The Halton numbers generated for first three dimensions using 2, 3, and 5 as the prime numbers for bases, respectively, for n ranging from 1 to 8 are shown in Table 1.

Table 1. Halton sequences for first 3 dimensions.

n	Dim 1 $p=2$	Dim 2 $p=3$	Dim 3 $p=5$
$n=1$	1/2	1/3	1/5
$n=2$	1/4	2/3	2/5
$n=3$	3/4	1/9	3/5
$n=4$	1/8	4/9	4/5
$n=5$	5/8	7/9	1/25
$n=6$	3/8	2/9	6/25
$n=7$	7/8	5/9	11/25
$n=8$	1/16	8/9	16/25

The samplings of two dimensional space $[0,1]^2$ with 500 points are shown for both Monte Carlo using random sequences in both dimensions; and Quasi Monte Carlo sampling using Halton sequences with p_n = 2 and p_n = 3 respectively in two dimensions are shown in Fig. 1. It is clear from the figure that the Halton sequence points sample the region more uniformly than the random sequences. The problem of clustering of points in Monte Carlo sampling is significantly reduced in Quasi Monte Carlo sampling.

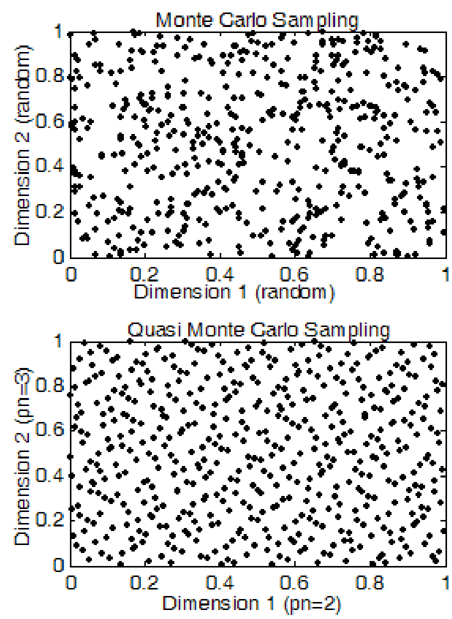

Fig. 1. Sampling of the two dimensional space using random and Halton sequences, respectively.

Another very important aspect of the Halton sequences, as is clear from table 1, is that the sequences in any two different dimensions are not the same, i.e., the sample points in a quasi-random sequence are, in a precise sense, “maximally avoiding” of each other. This property has been utilized in the Galerkin approach for the MoM solution of the radiation from wire antenna with the exact form of the kernel. The two dimensional problem in this approach leads to integration of four dimensional singular functions in which the integrands have line singularities. This problem of singularity can be removed automatically by the QMCI using Halton sequences with different bases for each dimension.

III. FORMULATION OF THE PROBLEM

The numerical modeling of a wire dipole antenna [9-14] is taken up as a test problem A dipole wire antenna of finite radius *a,* and length *l* is shown in Fig. 2. The wire has a finite thickness, but it is considered thin as $a << \lambda$. For a thin antenna, the unknown current varies on the surface only along the *z* axis and variation along the coordinate φ is negligible. Figure 2 shows a center fed dipole wire antenna of radius *a*, and length *l*. Both source and observation points are on the surface of the antenna characterized by both axial and circumferential coordinates. The wire antenna is usually formulated using two approaches, depending on whether the exact (full) kernel (EK) or the approximate (reduced) kernel (AK) of the integral equation is used. In the exact kernel formulation [15], where the kernel is singular, several singularity extraction and correction techniques have been used that require analytical methods.

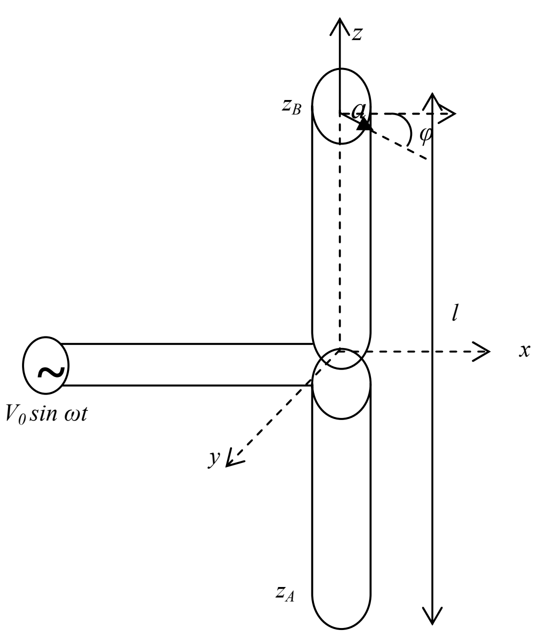

Fig. 2. A center fed dipole wire antenna of radius *a*, and length *l*.

Thus, the evaluation of matrix elements requires analytical pre processing of the kernel, before numerical integration, making the entire procedure very complicated.

The need to get rid of the singularity has resulted in the approximated or the reduced form of the kernel by Richmond [16]. The corresponding EFIE is,

$$-E_z^{\,i} = \frac{\lambda Z_0}{j8\pi^2}\int_l J_z(z')\frac{e^{-jkR}}{R^5}[(1+jkR)(2R^2-3a^2)+(kaR)^2]dz' \quad (4)$$

where $R=\sqrt{a^2+(z-z')^2}$ and $Z_0 = 337\Omega$, the free space impedance.

It is clear from equation (4) that both the observation and the source points are not on the surface, but the source coordinates are along the *z*-axis. The exclusion of the ϕ coordinates from the kernel is a very crude approximation of it, and therefore the reduced kernel formulation results in loss of one dimension of the problem.

In the exact kernel formulation, on the surface of the antenna, the EFIE takes the form,

$$-E^{inc}(z,\phi) = \frac{1}{j4\pi\omega\varepsilon_0}\int_{z_A}^{z_B} J(z')\left(\frac{\partial^2}{\partial z^2}+k^2\right)G(z,\phi;z'\phi')adz'\,d\phi' \quad (5a)$$

where

$$G(z,\phi;z',\phi') = \frac{e^{-jk\sqrt{(2a^2(1-\cos(\phi-\phi')+(z-z')^2}}}{\sqrt{(2a^2(1-\cos(\phi-\phi')+(z-z')^2}}. \quad (5b)$$

A. MoM Formulation

The unknown current on the surface of the antenna is expanded as,

$$J(z') = \sum_{n=1}^{M}\alpha_n f_n(z') \quad (6)$$

where α_n ; n = 1, 2, . . ., M; are unknown amplitudes to be determined.

The point matching technique leads to the matrix equation of the form,

$$-j4\pi\omega\varepsilon_0\begin{bmatrix} E_z^{inc}(z_1)\\ E_z^{inc}(z_2)\\ \cdot \\ \cdot \\ E_z^{inc}(z_M)\end{bmatrix} = [A_{mn}][\alpha_n] \quad (7)$$

with

$$A_{mn} = \int_{z_A}^{z_B} f_n(z')(\frac{\partial^2}{\partial z^2} + k^2)G(z_m, \phi; z'\phi')adz'd\phi' \cdot \tag{8}$$

And the Galerkin's technique leads to the matrix equation of the form,

$$-j4\pi\omega\varepsilon_0 \begin{bmatrix} \int_{z_A}^{z_B} E_z^{inc}(z)f_1(z)dz \\ \int_{z_A}^{z_B} E_z^{inc}(z)f_2(z)dz \\ . \\ . \\ \int_{z_A}^{z_B} E_z^{inc}(z)f_M(z)dz \end{bmatrix} = [A_{mn}][\alpha_n] \tag{9}$$

with

$$A_{mn} = \int_{z_A}^{z_B}\int_{z_A}^{z_B} f_n(z')G(z,\phi;z'\phi')(\frac{\partial^2}{\partial z^2} + k^2)f_m(z)a^2dz'd\phi'dzd\phi \cdot \tag{10}$$

B. MCI and QMCI Technique Implementation

The fact that the kernel of the EFIE is singular is evident from equation (5). For the point matching technique, the local correction technique is applied to deal with the singularity arising in the integration of equation (8) where the integrand diverges at the points where $\phi' = \phi_m$ and $z' = z_m$. Both the MCI and QMCI can be utilized for treating this singularity. This technique can be implemented by excluding small regions about the points of singularity ϕ_m and z_m when the random points are generated for the variables ϕ' and z' respectively by embedding the required condition directly in the program code employed for the purpose in simulation.

For Galerkin's technique, in the problem under investigation, QMCI is applied to remove the singularity arising in the integration of equation (10). This is a four dimensional integration in variables ϕ, ϕ', z and z', where the integrand shows line singularity at the points where $\phi = \phi'$ and $z = z'$. This problem of line singularity can be is done by generating quasi random Halton sequences with different bases for the variables ϕ, ϕ', z and z', since no two Halton sequences with different bases are same, the singularity in the kernel is taken care off automatically.

IV. NUMERICAL EXAMPLES

As a test example for the proposed technique, a half wavelength wire dipole antenna, fed at its center by a signal generator of frequency 850 MHz is considered. The corresponding wavelength is λ = 0.3529 m. The radius is a = 0.001λ. The two ends of the antenna have the coordinates $z_A = -\lambda/4 = -0.088235$ m; $z_B = \lambda/4 = 0.088235$ m.

Three cases for the problem have been studied:

Case I: The proposed exact kernel with the sinusoidal incident field modeling and implementation of MCI with sub domain pulse basis functions. The entire length of the antenna is divided into equal length segments and each segment is at least λ/10 in length. Further, the number of segments is odd, so that the feed gap is modeled as a single segment. Taking these two factors into consideration, the antenna is divided into 21 equal length segments and the point matching technique with mid points of each segment as the observation points is adopted. In this modeling, the diagonal terms of the matrix are from equation (8). The incident electric field is,

$$E_z^{inc}(z) = \sin(k(z - z_A)). \tag{11}$$

The total number of random points taken for the MCI is N = 5000.

Case II: The proposed exact kernel with the sinusoidal incident field in equation (11) and implementation of MCI with entire domain polynomial basis functions. Since the incident field is parallel to the z axis, from the geometry of the antenna, the z directed current will be zero at the edges perpendicular to z axis, i.e., at $z = z_A$ and $z = z_B$. The entire domain basis function employed is the polynomial,

$$J(z') = \sum_{even\ n=2}^{M} z'^n - (z_A)^n. \tag{12}$$

The simulation is carried out for M = 4, making number of terms in the expansion is equal to 2. It is observed that the results convergence for M > 4. The total number of random points taken for the MCI is N = 10000.

Case III: The reduced kernel with conventional quadrature integration and the delta gap source modeling with sub domain pulse basis functions. The antenna is divided into 21 equal length segments, with mid points of each segment as the observation points, as in case I. The simulation results of the normalized current distribution over the length of the wire antenna, obtained for the three cases mentioned above is plotted in Fig. 3. As is evident from the figure, all the three results are in excellent agreement, which justifies the effectiveness of the proposed method. The efficiency of the MCI techniques adopted for integration of the singular function used with the entire domain polynomial basis functions given by equation (9) is evident from Table 2. As can be seen, this combination results in reduction in storage requirements by more than 100 times, making the proposed technique very efficient.

Next, the efficiency of the QMCI technique is tested on two problems. First, a half wavelength wire dipole antenna, fed at its center by a signal generator of frequency 300 MHz is considered. The corresponding wavelength is λ = 1 m. The radius is a = 0.001λ. The two ends of the antenna have the coordinates z_A = -0.25m and z_B = 0.25m.

The results are obtained by QMCI implementation and sub domain pulse basis function with 21 segments using N = 500 for both point matching and Galerkin's approach and plotted in Fig. 4. The results are in very good agreement.

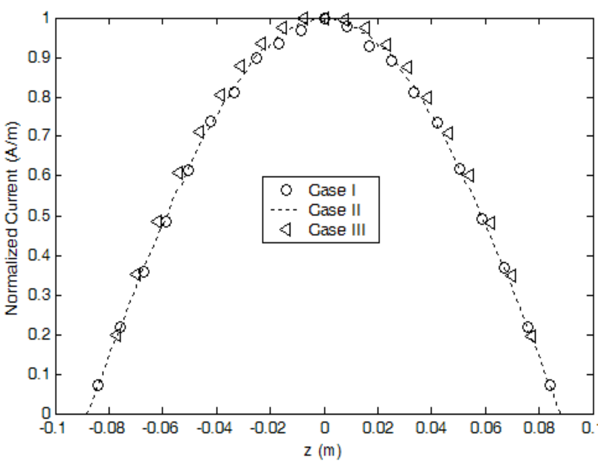

Fig. 3. The normalized current distribution on the wire dipole antenna (l= λ /2) for the three cases.

Table 2. A comparison of matrix size.

kernel	*Basis function*	*Integration technique*	*Matrix Size*
Exact	Sub sectional pulse	MCI N = 5000	21 x 21
Exact	Poly nomial, given by (12) with M = 4	MCI N = 10000	2 x 2
Reduced	Sub sectional pulse	Conventional	21 x 21

Secondly, a one wavelength wire dipole center fed antenna, of frequency 600 MHz is considered. The corresponding wavelength is λ = 0.5 m. The radius is a = 0.001λ. The two ends of the antenna have the coordinates z_A = -0.25m and z_B = 0.25m. The results are obtained by QMCI implementation and sub domain pulse basis function with 21 segments.

Figure 5 compares the both point matching and Galerkin's methods for QMCI implementation using N = 500 for one wavelength dipole which again shows a very good agreement between the two results. It is seen that QMCI is very efficient as is gives the desired results in only 500 quasi random points.

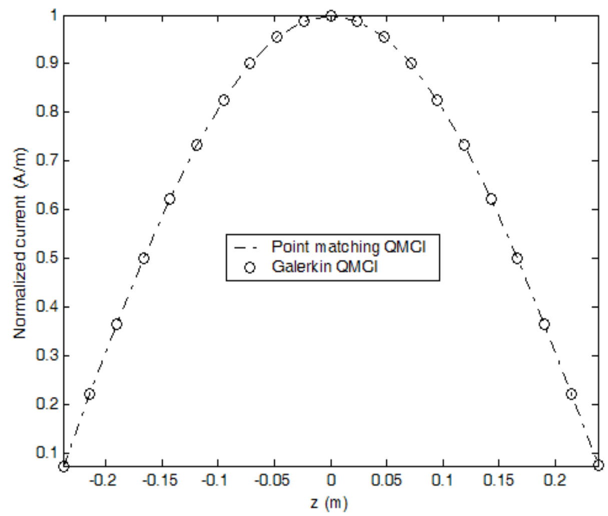

Fig. 4. The normalized current distribution on the wire dipole antenna (l= λ /2) using QMCI technique, f = 300 MHz.

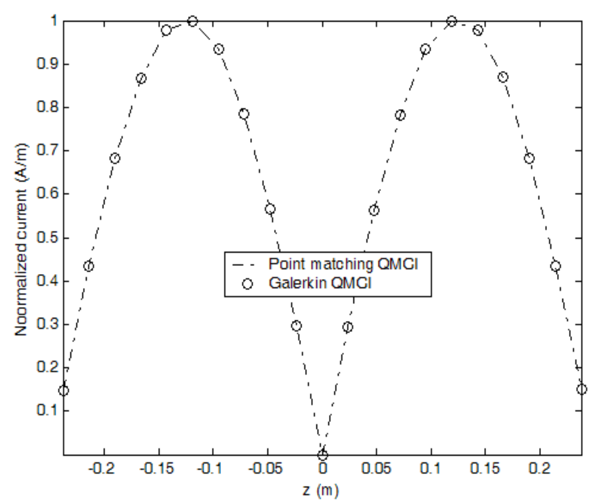

Fig. 5. The normalized current distribution on the wire dipole antenna (l= λ) using QMCI technique, f = 600 MHz.

V. CONCLUSION

The QMCI technique using Halton sequence is proposed in the MoM solution of the EFIE for some radiation problems. As an example, radiation from a simple wire antenna is investigated. It is found that the proposed technique not only solves the radiation problem efficiently but also takes care of the singularity problem appearing in the kernel of the integrand due to the inherent property of the Halton sequence.

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